GIBBS MEASURES OF NONLINEAR SCHRÖDINGER EQUATIONS AS LIMITS OF MANY-BODY QUANTUM STATES

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Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. We study the problem of the derivation of Gibbs measures as mean-field limits of Gibbs states in many-body quantum mechanics.

We present two approaches to this problem. The first one is based on a perturbative expansion in the interaction. This expansion is then analysed by means of Borel resummation techniques and a graphical representation. The second approach is based on a functional integral representation. The latter can be interpreted as a rigorous version of an infinite-dimensional stationary phase argument. This is joint work with Jürg Fröhlich, Antti Knowles, and Benjamin Schlein.