MORREY'S INEQUALITY IN DIRICHLET SPACES

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Morrey's inequality in $\mathbb{R}^n$ is a classical Sobolev embedding that has many important applications, for instance in the regularity theory of elliptic PDEs. Roughly speaking, this inequality asserts that functions in the Sobolev space $W^{1,p}(\mathbb{R}^n)$ are Hölder continuous for any $n < p < \infty$ with an explicit optimal exponent that depends on n and p.

In this talk we will present Morrey's inequality in the more general framework of Dirichlet spaces with (sub-)Gaussian heat kernel estimates. In particular, we will discover that the optimal exponent not only depends on the Hausdorff and the walk dimension, but also on a further invariant of the space. To this end, we will discuss a recent approach to $(1, p)$-Sobolev spaces via heat semigroups inspired by ideas that go back to work of de Giorgi and Ledoux.

If time permits, we will outline some results and conjectures concerning a critical exponent which might be related to other dimensions of interest in the theory of metric measure spaces.

This talk is based on joint work with F. Baudoin.