NON-UNIQUENESS OF BUBBLING FOR WAVE MAPS

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We consider wave maps from $\mathbb{R}^{2+1}$ to a smooth Riemannian manifold. Like solutions to many energy critical PDE, wave maps can develop singularities where the energy concentrates on arbitrarily small scales. At points of concentration, it is known that the wave map, suitably transformed, converges weakly to a harmonic map, also called a bubble. One fundamental question is whether this weak limit is unique.

We show by example that uniqueness may not hold if the target manifold is not analytic. In particular, we exhibit a continuum of different bubbles at the origin, each of which arise as the weak limit along a different sequence of times approaching the blow-up time. Our construction is inspired by Peter Topping's analogous example of a "winding" bubble in harmonic map heat flow, however, the Hamiltonian nature of the wave maps will necessitate different arguments. This is joint work with Max Engelstein (University of Minnesota).