BOUNDARY VALUE PROBLEMS FOR EINSTEIN METRICS

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Let M be a compact oriented d-dimensional manifold with boundary N. A natural geometric boundary value problem is to find an asymptotically hyperbolic Einstein metric g on (the interior of) M with prescribed 'conformal infinity on N. A little more precisely, the problem is to find (Einstein) g with the boundary condition that $x^2 g$ tends to a metric h on N as x goes to 0, x being a boundary defining function for N. The prototype is the hyperbolic metric g on the ball, with conformal infinity the round metric on the boundary sphere. Since the pioneering work of Graham and Lee (1991) the problem has attracted attention from a number of authors.

In this talk, I shall explain a gauge-theoretic approach to the problem which works in dimension $d = 4$, and explain how it can be used to obtain some new results for this boundary value problem. Based on joint work with Joel Fine and Rafe Mazzeo.