ON THE POINTWISE CONVERGENCE OF SOLUTIONS OF THE SCHRÖDINGER EQUATION TO THE INITIAL DATUM

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We consider solutions to the linear Schrdinger equation on \mathbb{R}^n with initial data in the Sobolev space H^s . A classic result of Carleson tells us that when n = 1c the condition $s \ge 1/4$ is sufficient for the solutions in order to converge almost everywhere to their initial data, as time goes to zero. Dahlberg and Kenig have then proved, giving explicit counterexamples, that solutions with less regular data may not converge (this is indeed the case in any dimension). Thus the threshold $s \ge 1/4$ has been conjectured to be the correct one even in higher dimensions. However, this conjecture has been recently disproved in a series of paper. We will present the Dahlberg-Kenig counterexample and a more sophisticated one (obtained in collaboration with K. Rogers) which improves it in higher dimensions. Then we will show how to combine both to prove that $s \ge n/(2n+2)$ is necessary to guarantee that any solution with data in H^s converges to its datum. This has been recently proved by Bourgain and has been shown to be also sufficient in dimension n = 2 by Du, Guth and Li and in higher dimensions by Du and Zhang.