UNEXPECTED COMBINATORIAL PROPERTY OF ALL PLANAR MEASURES

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Paraproducts are arguably the simplest singular operators. But the standard Calderón– Zygmund theory fails to apply in the product setting. On the other hand, in PDE the need for such multi-parameter SIO appears very naturally when one needs a Leibniz rule for non-homogeneous derivatives, say on product of two functions of two variables. Obviously it is an absolutely fundamental question. Another need for multi-parameter weighted paraproducts appear from complex analysis in polydisc as in Chang–Fefferman series of papers. While embedding Dirichlet space on polydisc into L^2 on torus with respect to a given measure, we stumble upon a counterintuitive property of planar measures that seems to be contradictory to Carleson quilts example in Chang-Fefferman theory. Of course it is not, and our property gives necessary and sufficient condition for boundedness of weighted bi-parameter paraproducts, thus solving the problem of embedding of Dirichlet space of analytic functions in bidisc.