Localization of eigenfunctions via an effective potential

We discuss joint work with Douglas Arnold, Guy David, Marcel Filoche and Svitlana Mayboroda. Consider the operator

$$L\,f = -\Delta f + V f$$

on the torus $\mathbb{R}^n/N\mathbb{Z}^n$, that is, with periodic boundary conditions and period N. The eigenfunctions of L are often localized, as a result of disorder of the potential V. In earlier work, Filoche and Mayboroda introduced the function u solving $Lu = 1$, and showed numerically that it strongly reflects this localization. Here, we deepen the connection between the eigenfunctions and this *landscape* function u by proving that its reciprocal $1/u$ acts as an *effective potential*. The effective potential governs the exponential decay of the eigenfunctions of the system and delivers information on the distribution of eigenvalues near the bottom of the spectrum.

The results are valid for variable coefficient operators on closed manifolds with or without boundary, independent of smoothness, but the difficulties are already present in this special case. The main issue is to make universal estimates, in particular, ones that are uniform in N as $N \to \infty$ for non-negative, bounded potentials V.