ON A CENTRAL LIMIT TYPE CONJECTURE FOR THE NODAL STATISTICS OF QUANTUM GRAPHS

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Understanding statistical properties of Laplacian eigenfunctions in general and their nodal sets in particular have an important role in the field of spectral geometry, and interest both mathematicians and physicists. A quantum graph is a system of a metric graph with a self-adjoint Schrödinger operator acting on it. In the case of quantum graphs it was proven that the number of points on which each eigenfunction vanishes, also known as the nodal count, is bounded away from the spectral position of the eigenvalue by a topological quantity, the first Betti number of the graph. A remarkable result by Berkolaiko and Weyand (with another proof for discrete graphs by Colin de Verdiere) showed that the nodal surplus is equal to a magnetic stability index of the corresponding eigenvalue.

Both from the nodal count point of view and from the physical magnetic point of view, it is interesting to consider the distribution of these indices over the spectrum. In our work we show that such a density exists and define a nodal surplus distribution. Moreover this distribution is symmetric, which allows to deduce the Betti number of a graph from its nodal count. A further result proves that the distribution is binomial with parameter 1/2 for a certain large family of graphs. The binomial distribution satisfies the Central limit theorem (CLT) and converge under appropriate normalization to normal distribution. A numerical study indicates that the CLT convergence is independent of the specific choice of the growing family of graphs. In my talk I will talk about our latest results extending the number of families of graphs for which we can prove the CLT convergence.

This talk is based on joint works with Ram Band (Technion) and Gregory Berkoliako (Texas A&M).