ORTHOGONAL POLYNOMIALS AND SHARP STRICHARTZ ESTIMATES

FELIPE GONÇALVES

Orthogonal polynomials have been used to produce sharp estimates in Harmonic Analysis in several instances. The first most notorious and original use was in Beckner's thesis [1], where he proved the sharp Hausdorff-Young inequality using Hermite polynomial expansions. More recently, Foschi [4] used spherical harmonics and Gegenbauer polynomials in his proof of the sharp Tomas-Stein adjoint Fourier restriction inequality for the sphere. Later on, this strategy was extended by Carneiro and Oliveira e Silva [3] for other dimensions and even exponents.

In this talk we discuss the recent work [5], where we develop an approach using Hermite and Laguerre polynomials to produce sharp Strichartz estimates for the Schrödinger equation for all even exponents (smoothing estimates were also produced in [2] using Gegenbauer polynomials). This approach was later exploited in [6] to produce a sharpened inequality in dimension 2 for radial initial data (where the distance to the extremizers is involved). We feel that the proof the latter inequality illustrates quite well the beauty and yet rigidity of these arguments, and it is here that we will focus our attention."

References

[1] W. Beckner, Inequalities in Fourier Analysis, Annals of Mathematics, 102 (1975), 159-182.

[2] N. Bez and M. Sugimoto, Optimal constants and extremizers for some smoothing estimates, Journal d'Analyse Mathmatiqu, vol. 131, issue 1, (2017), p. 159-187

[3] E. Carneiro and D. Oliveira e Silva, Some Sharp Restriction Inequalities on the Sphere, Int. Math. Res. Not., vol 2015, issue 17, p. 8233-8267.

[4] D. Foschi, Global maximizers for the sphere adjoint Fourier restriction inequality, Journal of Functional Analysis, vol. 268, issue 3, (2015), p. 690-702.

[5] F. Gonçalves, Orthogonal polynomials and sharp estimates for the Schrödinger Equation, Int. Math. Res. Not., vol. 2017, issue 00, p. 1-28.

[6] F. Gonçalves, A sharpened Strichartz inequality for radial functions, arXiv:1709.08100.

University of Alberta, Mathematical and Statistical Sciences, CAB 632, Edmonton, Canada T6G 2G1

E-mail address: felipe.goncalves@ualberta.ca

URL: sites.ualberta.ca/~goncalve

Date: February 13, 2018.