

## COMMUTATORS OF SINGULAR INTEGRALS AND MULTI-PARAMETER BMO

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We study the boundedness of commutators of singular integrals in connection with different types of John-Nirenberg spaces of bounded mean oscillation (BMO) in the multi-parameter setting. It is a classical result of Coifman, Rochberg, and Weiss that function  $b$  is in BMO, if and only if the commutator  $[b, R_j]$  is bounded on  $L^p$ ,  $1 < p < \infty$ , where  $R_j$  is a Riesz transform and  $b$  is understood as a pointwise multiplication operator. We will discuss several recent results along this line, including the discovery of a new class of BMO spaces in the multi-parameter setting, which are characterized by the boundedness of certain commutators of multi-parameter singular integrals. Such results have natural applications in weak factorization of Hardy spaces, Hankel operators, and div-curl estimates. One of the main tools that is involved is a representation of singular integrals as averages of dyadic operators.