RESONANCES FOR OPEN QUANTUM MAPS

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Quantum maps are a popular model in physics: symplectic relations on tori are quantized to produce families of $N \times N$ matrices and the high energy limit corresponds to the large N limit. They share a lot of features with more complicated quantum systems but are easier to study numerically. We consider open quantum baker's maps, whose underlying classical systems have a hole allowing energy escape. The eigenvalues of the resulting matrices lie inside the unit disk and are a model for scattering resonances of more general chaotic quantum systems. However in the setting of quantum maps we obtain results which are far beyond what is known in scattering theory.

We establish a spectral gap (that is, the spectral radius of the matrix is separated from 1 as $N \to \infty$) for all the systems considered. The proof relies on the notion of fractal uncertainty principle and uses the fine structure of the trapped sets, which in our case are given by Cantor sets, together with simple tools from harmonic analysis, algebra, combinatorics, and number theory. We also obtain a fractal Weyl upper bound for the number of eigenvalues in annuli. These results are illustrated by numerical experiments which also suggest some conjectures.

This talk is based on joint work with Long Jin.