THE LAPLACIAN ON PLANAR HILBERT SCHEMES

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The Hilbert scheme $\mathrm{Hilb}^n(\mathbb{C}^2)$ $(n \geq 2)$ of (unlabelled) sets of n points in $\mathbb{C}^2$ is a smooth complex manifold of dimension $2n$ containing the configuration space of distinct points as an open dense subset. Using hyper-Kähler reduction, Nakajima showed that $\mathrm{Hilb}^n(\mathbb{C}^2)$ has a hyper-Kähler metric and Carron showed that this was the same as the metric constructed by Joyce. It was conjectured by Vafa and Witten that the L^2 cohomology of $\mathrm{Hilb}^n(\mathbb{C}^2)$ is one-dimensional. Hitchin showed that there is no cohomology outside the middle dimension and Carron proved the result for $n = 2, 3$. In this talk I will describe the asymptotic behaviour of the space, the metric and the Laplacian; these may, indeed should, lead to a resolution of the conjecture.

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