Lattice point count and continued fractions

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In this talk I'll discuss a lattice point count for a thin semigroup inside $\mathrm{SL}_2(\mathbf{Z})$. It is important for applications that one can perform this count uniformly throughout congruence classes and for arbitrary moduli. The approach to counting is dynamical - with input from both the real and finite places. At the real place one brings ideas of Dolgopyat concerning oscillatory functions into play. At finite places, a rapid mixing property is supplied by expansion of Cayley graphs and injected into the thermodynamical formalism. The expansion of the relevant Cayley graphs was first established by Bourgain and Gamburd (for prime places) and extended to arbitrary moduli by Bourgain and Varjú. Until recently it was only known how to apply these expansion results in the thermodynamical setting for squarefree moduli. Recently (with Bourgain and Kontorovich) a new decoupling method was developed that allowed arbitrary moduli to be treated. As result of all this work, a power saving in the size of the exceptional set in Zaremba's conjecture is now available. This talk is based on joint works with Oh and Winter, and with Bourgain and Kontorovich.