# THE FUGLEDE CONJECTURE HOLDS IN $\mathbb{Z}_p \times \mathbb{Z}_p$

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Abstract: In the 70s Bent Fuglede conjectured that if $\Omega$ is bounded domain in $\mathbb{R}^d$, then $L^2(\Omega)$ has an orthogonal basis of exponentials if and only if $\Omega$ tiles $\mathbb{R}^d$ by translation. This idea led to much activity and this conjectured was established in a variety of instances, including that of convex domains in the plane by Iosevich, Katz and Tao in 2003. In 2004, Tao disproved the conjecture by exhibiting a set in $\mathbb{R}^5$ which does not tile, yet possessing an orthogonal basis of exponentials. In the subsequent years counter-examples in both directions were established by Kolountzakis, Matolcsi and others in dimensions 4 and higher. Most of these counter-examples were based on constructions in $\mathbb{Z}_p^d$ which were then transplanted into Euclidean space. It was widely believed that the counter-examples would eventually go down all the way to dimension 1 where the Fuglede conjecture has been previously tied by Laba and others to some very interesting number theoretic questions. However, in this talk we are going to show that the Fuglede conjecture holds in $\mathbb{Z}_p^2$ if $p$ is a prime. The proof is a blend of elementary Fourier analysis and basic algebra.