SPUR 2010 ABSTRACTS

Aldo Pacchiano Camacho

Title: Fast optical character recognition via cryptanalysis and geometry

Mentor: Kartik Venkatram

The fundamental task facing Optical Character Recognition (OCR) systems is the conversion of input document images into corresponding sequences of symbolic character codes. Traditional OCR systems tackle this problem by performing a bottom up analysis. The image of each symbol is isolated, then classified based on its pixel intensities and finally classified based on its shape. While such shape based classifiers are initially trained on an enormous array of fonts, their performance decreases notably when a new type of font is presented. In this paper we attempt to bypass this problem by pursuing a cryptographic codebreaking approach. Along the way we present the advantages and disadvantages of such a method, discuss some algorithms on skeletal finding and discuss different an OCR system that combines simple geometric checks and cryptogram decoding.

Harrison Chen

Title: Representation of rational Cherednik algebras in positive characteristic

Mentor: Martina Balagović

The rational Cherednik algebra H_c is an infinite-dimensional algebra over a field k that is associated with a reflection group G and its reflection representation $\mathfrak{h}$. Though the rational Cherednik algebra has been studied extensively for the past fifteen years, it has mostly been studied in the case when $k = \mathbb{C}$. In this paper, we study the case when k is an algebraically closed field of characteristic p. We are particularly interested in groups for which p divides the order, such as $GL_n(\mathbb{F}_p)$, since these groups cannot be lifted to characteristic zero. For an irreducible G-representation τ, we define an analogue of the Verma module $M_c(\tau)$ from Lie theory. There is a contravariant form B defined on Verma modules.

The object of interest in our study are $L_c(\tau) = M_c(\tau)/\ker(B)$. We prove that these constitute all finite-dimensional graded irreducible representations of H_c. In particular, we are interested in computing the character of $L_c(\tau)$, which we define as its Hilbert series. We develop some general methods for computing the character. We then turn to a few specific cases, in which we used the Magma computer algebra system to compute characters for a few examples and formulate conjectures. This led to a full computation of the character of such modules in the case when $G = GL_n(\mathbb{F}_q)$ and $G = SL_n(\mathbb{F}_q)$ when $q = p^r$ and τ is the trivial representation.

Andrei Frimu

Title: On The Maximal Number Of Rational Points On A Curve Of Genus 4 over $\mathbb{F}_{11}$ and $\mathbb{F}_{13}$

Mentor: Bhairav Singh

In this paper we investigate the maximum number of rational points on smooth, projective, absolutely irreducible genus 4 curves over the finite fields $\mathbb{F}_{11}$ and $\mathbb{F}_{13}$. The best known upper bounds are 34 points for $\mathbb{F}_{11}$ and 39 points for $\mathbb{F}_{13}$. We show that a curve defined by a biquadratic equation over $\mathbb{F}_{11}$ has at most 32 points and such a curve over $\mathbb{F}_{13}$ has at most 38 points. We construct examples with these number of points. We find restrictive conditions for the curves with more points than our examples.

Raghu Mahajan

Title: Sweepouts and level-set flow

Mentor: Lu Wang

In this note, we use level-set flow to prove the existence of simple closed geodesics on a 2-dimensional Riemannian manifold that is diffeomorphic to the 2-sphere. We show that the level-set with maximum length comes arbitrarily close to being a simple closed geodesic at large times. We also investigate the behavior of sweepouts under level-set flow.

Christopher Policastro

Title: Category $\mathcal{O}$ for the Rational Cherednik Algebra of G_{12}

Mentor: Martina Balagović

The rational Cherednik algebra of a complex reflection group W with reflection representation $\mathfrak{h}$ is defined as a deformation of the algebra $\mathbb{C}[W] \ltimes S(\mathfrak{h}^* \oplus \mathfrak{h})$ depending on certain complex parameters. In this paper, we describe the irreducible representations in category $\mathcal{O}$ of the rational Cherednik algebra associated to G_{12}, in the Shephard-Todd notation, for an arbitrary complex parameter. In particular, we determine semisimplicity conditions on the category, Grothendieck group expressions for irreducible modules, and the characters for each irreducible finite dimensional representation. The determination of the structure of $\mathcal{O}$ uses computational and algebraic methods that should be easily applicable to other two dimensional complex reflection groups in the case of equal parameters.

Jonathon Schneider

Title: Face numbers and Ehrhart polynomials

Mentor: Hoda Bhidkori

In this paper, we explore several statistics of certain families of hyperplane arrangements. Following the work of Athanasiadis in his thesis, we compute the Whitney polynomial and f-vector for the Catalan arrangement and the generalized Catalan arrangement. Using a theorem of Stanley, we also compute a general form of the Ehrhart polynomial of the dual zonotope for a class of cones over rational deformations of the braid arrangement. We apply this form to obtain specific results for the Ehrhart polynomials associated with the semi-order and Catalan arrangements. Finally, we investigate the properties of hyperplane arrangements with specific underlying graphical structures (trees and cycles) and obtain some surprisingly general results.

XiaoLin Shi

Title: Combinatorial Properties and Optimization of a Rooted Graph Polynomia

Mentor: Amanda Redlich

Let G be a rooted graph where each edge is independently present with probability p. We consider the expected number of vertices in the connected component of G containing R. This expected value $EV(G;p)$ is a polynomial of p. In this paper, we analyze combinatorial properties of this polynomial and consider various optimization problems that arise from the polynomial. We present a reconstruction result for rooted graphs with a unique cycle. It has been proved that G is k-edge connected if and only if $EV^{k-1}(G;1) = 0$. We give a more focused study of $EV'(G;p)$, relating $EV'(G;1)$ to the tree structure of 2-edge connected subgraphs of G. We also give an upper bound on the size of the 2-edge connected component of G containing R using $EV'(G;1)$ and $EV(G;1)$. In studying optimality of a rooted pan graph, we prove that the optimal root shifts continuously along the graph as p increases from 0 to 1. We conclude by stating further results when the edge probabilities are modified by assigning orientations to undirected edges or by assigning numbers to vertices.

Umut Varolgunes

Title: The dimension, the degree, and the initial complex of the secant ideal of the second hypersimple

Mentor: Hoda Bhidkori

Sullivant finds a Grobner basis for the second secant ideal of the second hypersimplex with respect to a circular term order. The circular term order and consequently this basis have a nice combinatorial structure. We use this basis to combinatorially analyze the initial complex of the ideal.

We first compute the dimension of its variety. Then we work on the degree of the variety, and rephrase the problem as a simple counting problem. Finally, we show that the initial complex is pure by combinatorial methods, which is also a corollary of an algebraic geometric theorem proven by Sturmfelds et al.

Sally Wolfe

Title: Cycle structure and pattern avoidance of *abc*-permutations

Mentor: Amanda Redlich

We define *abc*-permutations to be elements of S_n obtained by partitioning $[n]$ into three blocks of length a, b, c and exchanging the first and last blocks. In 2008, Pak and Redlich proved that the probability that an *abc*-permutation of length n is a long cycle goes to $6/\pi^2$ as n goes to infinity. We generalize this result, proving that an *abc*-permutation has k cycles, corresponding to the residue classes modulo k, where $k = \gcd(a+b, b+c)$. We derive a formula for the probability $\mathbf{p}_k(n)$ that an *abc*-permutation of length n has k cycles, and prove that $\mathbf{p}_k(n)$ goes to $(1/k^2)(6/\pi^2)$ as n goes to infinity.

We then characterize *abc*-permutations in terms of pattern avoidance, and explore families of permutations avoiding similar sets of patterns.

Qiaochu Yuan

Title: Lattice paths and the quadratic coefficient of Kazhdan-Lusztig polynomials

Mentor: Bhairav Singh

In 1979, Kazhdan and Lusztig defined a family $P_{u,v}(q)$ of polynomials indexed by pairs of elements of a Coxeter group W that have proven to be fundamental objects of study in representation theory. At the same time, they can be defined combinatorially, and so have also been studied by combinatorialists. Although it is now known that $P_{u,v}(q)$ depends only on the structure of the Bruhat interval $[u, v]$ as an abstract poset, explicit formulas which exhibit this invariance are only known in general for intervals of length at most 4. In this paper we use a formula of Brenti to give an explicit formula for the quadratic coefficient of $P_{u,v}(q)$ which is almost combinatorially invariant, and use this formula to give a combinatorially invariant formula for intervals of length at most 6 in the special case that $u = e$.