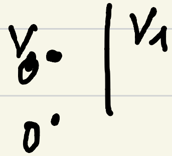
1)

Let V be a vector space of dim. n over F_q.
Let $\tilde{\mathcal{B}}$ be the set of sequences $V_0, V_1, \dots, V_{n-1}$ of mutually disjoint affine subspaces of V of dimensions $0, 1, 2, \dots, n-1$ respectively such that $V_0 \neq 0$ and V_i is parallel to the linear subspace OV_{i-1} of V spanned by V_{i-1} ($i = 1, \dots, n-1$).
(A subset X of V is said to be an affine subspace if it is of the form $v + X_0$ with $v \in V$, X_0 a linear subspace. Two affine spaces X, X' are said to be parallel if the corresponding linear subspaces coincide.)

For $n=2$ $\tilde{\mathcal{B}}$ consists of all



Let $G = GL(V)$. Now G acts on $\tilde{\mathcal{B}}$ by $g(V_0, \dots, V_{n-1}) = (gV_0, \dots, gV_{n-1})$. This action is transitive.
Let $\tilde{\mathcal{F}}$ be the vector space of functions $\tilde{\mathcal{B}} \to K$.
[K algebraic closure of F_q.]

2]

G acts on $\tilde{\mathcal{F}}$ by $g: f \to f'$ where
$f'(V_0, \dots, V_{n-1}) = f(g^{-1}V_0, \dots, g^{-1}V_{n-1})$.
We define an action of $(F_q^*)^n$ on $\tilde{\mathcal{F}}$ by
$(\lambda_0, \dots, \lambda_{n-1}) : (V_0, \dots, V_{n-1}) = (\lambda_0 V_0, \lambda_1 V_1, \dots, \lambda_{n-1} V_{n-1})$.
This action commutes with the G-action. It induces an action of $(F_q^*)^n$ on $\tilde{\mathcal{F}}$ by
$(\lambda_0, \dots, \lambda_{n-1}) : f \to f'$, $f'(V_0, \dots, V_{n-1}) = f(\lambda_0^{-1} V_0, \dots, \lambda_{n-1}^{-1} V_{n-1})$
This commutes with the action of G on $\tilde{\mathcal{F}}$. We have a direct sum decomposition
$\tilde{\mathcal{F}} = \bigoplus_\gamma \tilde{\mathcal{F}}_\gamma$ where γ runs over $(\mathbb{Z}/(q-1)\mathbb{Z})^n$
and $\tilde{\mathcal{F}}_\gamma = \left\{ f \in \tilde{\mathcal{F}},\ (\lambda_0, \dots, \lambda_{n-1}) f = \lambda_0^{\gamma_0} \lambda_1^{\gamma_1} \cdots \lambda_{n-1}^{\gamma_{n-1}} f \ \forall (\lambda_0, \dots, \lambda_{n-1}) \in (F_q^*)^n \right\}$.

(This is a general property of a linear repres. of $(F_q^*)^n$ on a finite dim. K-vector space.)
For example $\tilde{\mathcal{F}}_0$ can be identified with $\mathcal{F}$ as a repres. of G.
For any $\gamma \in (\mathbb{Z}/(q-1)\mathbb{Z})^n$ let J_γ be the set of all $i \in \{1, \dots, n-1\}$ such that $\gamma_i = \gamma_i + 1$.

3)

One can show:

There is a bijection {isom. classes of irred. reps. of G over K} $\longleftrightarrow$ {pairs (γ, J), where $\gamma \in (\mathbb{Z}/(q-1)\mathbb{Z})^n$, $J < J_\gamma$}. The irred. reps corresp. to $(\gamma\, J)$ with fixed γ are exactly the irred. subrepres. of $\widetilde{\mathcal{F}}_\gamma$. They are obtained as images of some linear maps analogous to $\Theta_J : \mathcal{F} \to \mathcal{F}$.

For example, if γ is non-degenerate in the sense that $\gamma_0, \dots, \gamma_{n-1}$ are mutually distinct, then there is a unique irreducible G-subrepres. of $\widetilde{\mathcal{F}}_\gamma$ (It corresponds to $(\gamma, \emptyset)$). It is the image of $T : \widetilde{\mathcal{F}}_{\gamma'} \longrightarrow \widetilde{\mathcal{F}}_\gamma$ where $\gamma' = (\gamma_{n-1}, \dots, \gamma_1, \gamma_0)$ and $(Tf)(V_0, \dots, V_{n-1}) = \sum f(V_0', \dots, V_{n-1}')$ sum over all $(V_0', \dots, V_{n-1}') \in \widetilde{\mathcal{B}}$ such that $V_0' \cap V_{n-1} \neq \emptyset$, $V_1' \cap V_{n-2} \neq \emptyset$, $\dots$, $V_{n-1}' \cap V_0 \neq 0$. (Each of these intersections is exactly one point.)

The total number of isom. classes of irred reps. of G/K is equal to $q^{n-1}(q-1)$.

4)

Assume that $n = 2$. The irred. G-subrep. which are subrep. of some $\mathcal{F}_\gamma$ ($\gamma_0 \neq \gamma_1$) have dimension d where d can be any integer in $2, 3, \dots, p-1$. The irr. G-subrep. which are subrep of $\mathcal{F}_\gamma$ ($\gamma_0 = \gamma_1$) are two in number; they have dim. $1, p$.