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Title: Using Fourier analysis to study diophantine equations

Abstract: Over the last few years, Jean Bourgain and Ciprian Demeter have developed a new theory in Fourier analysis, called decoupling. As one application, this method gives new bounds on the number of solutions to some diophantine equations, resolving a question of Vinogradov from the 1930s.

Fourier analysis was initially developed to study partial differential equations from physics, such as the heat equation and the wave equation. In the early 20th century, mathematicians realized that it was also connected to problems in number theory. In particular, the circle method, developed by Hardy, Littlewood, and Vinogradov, used Fourier analysis to estimate the number of solutions to diophantine equations such as $a_1^3 + a_2^3 + ... + a_s^3 = B$ with the variables in given ranges. The number of solutions to such an equation can be written exactly as a complicated integral involving a sum of complex exponentials with different frequencies. In order to estimate the number of solutions, one has to estimate the amount of cancellation in the sum. The issue of estimating cancellation between waves with different frequencies is also a central topic in studying the wave equation. Over the last few decades, Bourgain, Tom Wolff, and others have developed a geometric approach to this problem, and this geometric approach led to the decoupling theory.