February 24, 2016: David Vogan (MIT), "Galois cohomology and θ cohomology for real groups (following Adams)"

Suppose G is a complex algebraic group defined over $\mathbb{R}$; that is, endowed with an antiholomorphic action of $\Gamma = \mathrm{Gal}(\mathbb{C}/\mathbb{R})$. Then the Galois cohomology set $H^1(\Gamma, G)$ is defined; it is related to other real forms of G.

Let θ be a Cartan involution for $G(\mathbb{R})$. Then θ defines an algebraic action of $\mathbb{Z}/2\mathbb{Z}$ on G, and so a group cohomology set $H^1(\mathbb{Z}/2\mathbb{Z}, G)$. This cohomology lives in the world of *complex* algebraic groups, and so is a simpler thing than Galois cohomology. (In particular, it is computable by the `atlas` software.)

Adams proves that there is a natural bijection $H^1(\Gamma, G) \simeq H^1(\mathbb{Z}/2\mathbb{Z}, G)$, and draws a series of interesting consequences.