March 11, 2015: David Vogan (MIT), *Coherent sheaves on nilpotent cones.*

Suppose G is a complex reductive algebraic group, and $\mathcal{N} \subset \mathfrak{g}^*$ is the nilpotent cone. A conjecture of Lusztig, proved by Bezrukavnikov, says that there is a natural bijection

$$\text{irr. } G\text{-eqvt vector bdles on } G \text{ orbits on } \mathcal{N} \longleftrightarrow \text{dom. weights for } G.$$

(The coherent sheaves in the title arise because the left side is more or less obviously a basis for the Grothendieck group of G-equivariant coherent sheaves on $\mathcal{N}$.)

In the case of $SL(2)$, the dominant weights are non-negative integers, and the bijection is

$$0 \longleftrightarrow \text{trivial bundle on the regular orbit}$$
$$1 \longleftrightarrow \text{nontrivial bundle on the regular orbit}$$
$$p \longleftrightarrow (p-1)\text{-dimensional representation of } G \text{ at } 0 \quad (p \geq 2).$$

This bijection was computed explicitly in the case of $GL(n)$ by Achar in his 2001 thesis; it has not been computed completely for any other infinite series of groups.

I'll explain a definition and computation of this bijection in terms of finite-dimensional representation theory (thank you, Roman!); applications to infinite-dimensional representations that would follow from computing it; and possible generalizations to real and p-adic reductive groups.