

September 12, 2012: The theory of Kazhdan-Lusztig polynomials describes the singularities of certain orbit closures \overline{X} . One of the most famous of these singularities is that of the four-dimensional Schubert variety for $GL(4)$ corresponding to the permutation (3412) along the one-dimensional Schubert variety corresponding to (1324). This singularity (like any singularity of Schubert varieties) is measured by a Kazhdan-Lusztig polynomial: the larger the KL polynomial, the worse the singularity. In this example the polynomial is $q + 1$, in some sense the “first” example of a KL polynomial not equal to 1. (There is an industry of describing “nice” Schubert varieties in terms permutations avoiding “patterns” like (3412).)

I’ll describe a class of examples involving KL polynomials for real groups, in which the algorithm for computing the polynomials has some serious difficulty, and correspondingly the KL polynomial is particularly complicated. The first of these concerns a local system on the four-dimensional open orbit of $GL(4)$ on the flag variety for $Sp(4)$, along a $\mathbb{C}\mathbb{P}^1$ in the closure. In this case the KL polynomial is q , the first example of a KL polynomial with constant term zero. In the case of the split real group of type F_4 , the singularity (of a local system on a 24-dimensional variety along a 10-dimensional subvariety) has KL polynomial $q^6 + q^5$: this is the KL polynomial of highest degree for F_4 , and also the one vanishing to highest order at zero. I’ll say a little about the representation-theoretic meaning of these things (which looks interesting, but which I understand hardly at all).