October 31, 2012: Spherical varieties form a remarkable class of algebraic varieties equipped with an action of a complex reductive group G. They include toric, flag and symmetric varieties. Smooth affine spherical varieties are the local models of multiplicity free (real) Hamiltonian manifolds. A natural invariant of an affine spherical variety X is its weight monoid $\Gamma(X)$. It is the set of irreducible representations (or dominant weights) of G that occur in the coordinate ring of X, which is a multiplicity free G-module. In the 1990s F. Knop conjectured that it is a complete invariant for smooth affine spherical varieties, and in 2006 I. Loseu proved this conjecture. When G is non-abelian, little is known about the image of the map that sends a smooth affine spherical variety to its weight monoid. I will discuss joint work with G. Pezzini (Erlangen-Nürnberg) in which we combinatorially characterize those free and "G-saturated" monoids that belong to this image.