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Kazhdan-Lusztig polynomials, Soergel bimodules and some Hodge theory

Hecke algebras and their Kazhdan-Lusztig basis play a fundamental role in Lie theory. I will recall their definition and then describe Soergel bimodules, which provide a categorification of the Hecke algebra. Soergel came up with his definition with the geometry of Schubert varieties in mind, however his definition makes sense for any Coxeter group. Soergel conjectured that the indecomposable Soergel bimodules categorify the Kazhdan-Lusztig basis. His conjecture implies both the Kazhdan-Lusztig positivity conjecture and the Kazhdan-Lusztig conjecture on characters of simple highest weight modules for a complex semi-simple Lie algebra. The main goal of my talks will be to try to explain how some ideas from Hodge theory (in particular work of de Cataldo and Migliorini) can be transported to the setting of Soergel bimodules to yield a proof of Soergel's conjecture. The upshot is that Soergel bimodules have remarkable Hodge theoretic properties: they look like the real cohomology of smooth projective varieties. This is joint work with Ben Elias.