September 26: David Vogan, "Single-petaled K-types and Weyl group representations (after Hiroshi Oda)."

Suppose $G = KAN$ is a linear real reductive group, and M is the centralizer of A in K. Hiroshi Oda defines a representation of K to be For each simple restricted root α of A in $\mathfrak{g}$, choose a "root homomorphism" ϕ_α from $\mathfrak{sl}(2, \mathbb{R})$ to $\mathfrak{g}$, and define Z_α to be the image of i times the standard generator for the Lie algebra $\mathfrak{so}(2)$. (This element has integer eigenvalues in any representation of K.)

Suppose (σ, V) is an irreducible representation of K. Define

$$V_0 = \{v \in V^M \mid \sigma(Z_\alpha)(\sigma(Z_\alpha)^2 - 4)v = 0 \ (\alpha \in \Delta(\mathfrak{g}, A))\}.$$

(This is the part of V^M where Z_α generates an action with eigenvalues just 0 and ± 2.) Oda calls σ *quasi-single-petaled* if $V_0 \neq 0$. The space V_0 carries a representation σ_0 of the restricted Weyl group. The simplest example is $\sigma = triv$ equal to the trivial representation K; in that case $triv_0$ is the trivial representation of W.

The Chevalley restriction theorem relates the occurrence of the trivial representation of K in $S(\mathfrak{p})$ to the occurrence of the trivial representation of W in $S(\mathfrak{a})$. I'll explain Oda's generalization relating the occurrence of σ in $S(\mathfrak{p})$ to the occurence of σ_0 in $S(\mathfrak{a})$.

Dan Barbasch shows that the action of the standard intertwining operators for a spherical principal series on a quasi-single-petaled K-type σ can be related to those for Iwahori Hecke algebras and the Weyl group representation σ_0; in this way he is able to relate unitarity problems for real and p-adic groups. I'll try to explain what the Barbasch and Oda results have to do with each other.