DISCUSSION SECTION 11

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Abstract. Below you'll find a collection of questions. You should think of these questions less as a homework assignment and more as a playground to run around in and spark your imagination. It's more fun playing on the swings with friends, so come to discussion section to talk with others about anything you find confusing or exciting!

The following is [3, Rmk. 4.3.6].

Question 1. Let $\bar{X}$ be an n-dimensional singularity datum. Say

$$\bar{X} = (\{X_i\}, \{\zeta_i\}, \{p_i \colon E_i \to X_i\})$$

Show that a $\bar{X}$-manifold of dimension m is determines a stratified space

$$M_n \subseteq M_{n-1} \subseteq \cdots \subseteq M_0 = M$$

so that each open stratum $M_k - M_{k-1}$ is a smooth $(m-k)$-manifold with (X_k, ζ_k)-structure. What conditions are there on how the strata are glued together?

Question 2. If I wanted the stratification

$$M_n \subseteq M_{n-1} \subseteq \cdots \subseteq M_0 = M$$

to be of the form

$$M_k = M_{k-1} \sqcup N_k$$

for all k and some $(m-k)$-manifolds N_k, what conditions does the singularity datum $\bar{X}$ have to satisfy?

Question 3. Is it possible to define an n-dimensional singularity datum $\bar{X}$ (you choose the n) so that $\mathbb{R}^m$ with it's stratification

$$\mathbb{R}^1 \subset \mathbb{R}^2 \subset \cdots \subset \mathbb{R}^m$$

is a $\bar{X}$-manifold?

Recall the definition of a Whitney stratified space. You can do this by checking out the Wikipedia page on Whitney conditions or these notes from a course David Nadler taught.

Question 4. Are the stratified spaces coming from $\bar{X}$-manifolds Whitney stratified spaces?

Recall the definition of conically stratified spaces; see Definition 1.2 of Peter's notes or [2, Defn. A.5.5].

Question 5. Are the stratified spaces coming from $\bar{X}$-manifolds conically stratified spaces?

If you're feeling really bold, recall the definition of conically smooth stratified spaces from [1].

Question 6. Are the stratified spaces coming from $\bar{X}$-manifolds conically smooth stratified spaces?

Question 7. Write out in detail what a 1-dimensional singularity datum is. What is the data of a closed $\bar{X}$-manifold?

Question 8. Let $\bar{X}$ be a 1-dimensional singularity datum,

$$\bar{X} = (\{X_i\}, \{\zeta_i\}, \{p_i \colon E_i \to X_i\})$$

Let $\tilde{X}_0$ be the double cover associated to ζ_0. Assume $\pi_j \tilde{X}_0 = 0$ for $j > 0$.

Show that a closed $\bar{X}$-manifold determines a Feynman diagram.

If you don't have a definition of "Feynman diagram" in mind, draw pictures of the data of a closed $\bar{X}$-manifold and compare your pictures to the types of pictures that come up when you google image search "Feynman diagram."

References

[1] David Ayala, John Francis, and Hiro Lee Tanaka. Local structures on stratified spaces. *Adv. Math.*, 307:903–1028, 2017.

[2] Jacob Lurie. Higher algebra.

[3] Jacob Lurie. On the classification of topological field theories. In *Current developments in mathematics, 2008*, pages 129–280. Int. Press, Somerville, MA, 2009.