Goerss-Hopkins obstruction theory

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The basic cast of characters

This is a talk on chromatic homotopy theory, so:

- Fix a prime $p$ and a height $h$.
- $k$ is a perfect field of characteristic $p$.
- $\Gamma$ is a formal group law of height $h$ over $k$. 
Automorphisms of formal group laws

Last time, Lucy told us about the Morava stabilizer group

\[ \mathbb{G} = \text{Aut}(k, \Gamma) = \text{Aut}(\Gamma) \rtimes \text{Gal} \]

of automorphisms of the formal group law \( \Gamma \).
We also have Lubin-Tate theories.

We briefly recall their construction:

▶ Lubin-Tate: the formal group law $\Gamma$ admits a universal deformation to a formal group law over $W(k)[[u_1, \ldots, u_{h-1}]]$.

▶ This is Landweber flat.

▶ Hence, the Landweber exact functor theorem gives a even-periodic homology theory or spectrum $E(k, \Gamma)$. 
By construction, the Morava stabilizer group $\mathcal{G}$ acts on

$$\pi_* E(k, \Gamma) = W(k)[u_1, \ldots, u_{h-1}][u^{\pm 1}].$$

**Question**

*Does this lift to an action of $\mathcal{G}$ on the spectrum $E(k, \Gamma)$?*
Why might we want an action of $G$ on $E := E(k, \Gamma)$?

**Theorem (Devinatz-Hopkins)**

$E^h G \simeq S^0_{K(h)}$. 
Let $F < \text{Aut}(\Gamma)$ be a maximal finite subgroup. Then $EO_h := E^{hF}$ is a higher real $K$-theory.

You can also get localizations of $\text{TMF}$.

Also, we expect that $S^0_{K(h)} \simeq E^{h\mathbb{G}}$ can be approximated in terms of $E^{hF}$ for various finite subgroups $F < \mathbb{G}$.

When $h = 1, 2$, there exist resolutions of $S^0_{K(h)}$ by certain $E^{hF}$.

Moreover, the homotopy groups of $E^{hF}$ are amenable to calculation for example via homotopy fixed point spectral sequences.
We want to lift an action on homotopy groups to an action on spectra.

This problem is hard!
Solution?

“Make the problem harder!”
Making the problem harder

We refine the problem of lifting the action of \( G \) to the spectrum \( E \) in two ways:

1. Demand that \( G \) acts on \( E \) via \( E_\infty \)-ring automorphisms.
2. Study the entire moduli space of \( E_\infty \)-ring realizations of \( E \), i.e.,

\[
\mathcal{M}_{E_\infty}(E) = \left\{ \begin{array}{l}
\text{\( E_\infty \)-rings } A \text{ such that } A \simeq E \\
\text{as homotopy associative rings}
\end{array} \right\}.
\]

Remark

Implicit in (1) is the claim that \( E \) is an \( E_\infty \)-ring. A priori, we only know that \( E \) is a homotopy commutative ring from the Landweber construction. Most of the work will be showing that it is in fact \( E_\infty \).
The Goerss-Hopkins-Miller theorem

\[ \mathcal{M}_{E_\infty}(E) = \{ E_\infty \text{-rings } A \simeq E \} \]

Our main theorem today:

Theorem (Goerss-Hopkins-Miller)

\[ \mathcal{M}_{E_\infty}(E) \simeq BG. \]

- In other words, there exists a unique \( E_\infty \)-ring structure on \( E \), and \( \text{Aut}_{E_\infty}(E) \simeq \mathbb{G} \).
- Our tool to prove this is Goerss-Hopkins obstruction theory.
Other approaches, or a history of Juvitop

There have been other approaches to endow Morava $E$-theory with additional structure.

- A precursor is the Hopkins-Miller theorem, which studied $\mathbb{A}_\infty$-ring structures on $E$.
  - Danny gave a Juvitop talk on this in Fall 2016!

- There are other $\mathbb{E}_\infty$-obstruction theories developed to study Morava $E$-theory, e.g., Robinson’s $\Gamma$-homology.
  - Juvitop talk, Spring 2017!

- More recently, Lurie gave a different construction of Lubin-Tate theory as the solution to a moduli problem in spectral algebraic geometry. The $\mathbb{E}_\infty$-structure in this case is automatic.
  - Hood gave a Juvitop talk on this in Spring 2018!
Outline for the technical part of this talk

Primer on synthetic spectra

Goerss-Hopkins obstruction theory

Application to Morava $E$-theory
Why synthetic spectra?

We’ll follow a modern presentation of Goerss-Hopkins obstruction theory set out by Hopkins-Lurie and Pstrągowski-VanKoughnett, which situates moduli problems for commutative ring spectra in the context of **synthetic spectra**.

▶ Synthetic spectra is a technique for working with *resolutions*.

**Slogan**

Goerss-Hopkins theory is $(\mathbb{F}_\infty)$-Postnikov tower theory internal to synthetic spectra.
Definition of synthetic spectra

Let $E$ be an Adams-type homology theory (think Morava $E$-theory).

Let $\mathcal{S}p^\text{fp}_E$ be the category of finite spectra $P$ such that $E_* P$ is a finitely generated projective $E_*$-module.

Equip $\mathcal{S}p^\text{fp}_E$ with a Grothendieck topology where $\{P \to Q\}$ is a covering iff $E_* P \to E_* Q$ is surjective.

Definition

A (hypercomplete connective) synthetic spectrum is a product-preserving hypercomplete sheaf of spaces on $\mathcal{S}p^\text{fp}_E$.

Denote the category of synthetic spectra by $\mathcal{S}yn_E$. 
Basic features of $\text{Syn}_E$

- $(\text{Syn}_E, \otimes, 1)$ is a symmetric monoidal category, where
  - $\otimes$ is given by Day convolution, and
  - the monoidal unit $1$ is the *synthetic analogue*
    
    $\nu S^0_E := L(\text{Map}_{\text{Sp}}(-, S^0_E))$ of the $E$-local sphere.

- There is an autoequivalence of $\text{Syn}_E$, compatible with the symmetric monoidal structure, defined by sending $X$ to $X[1] = X \circ \Sigma^{-1}$.

  This gives rise to a second grading on $\text{Syn}_E$. 
The comparison map \( \tau \)

- There is a map

\[
\tau : \Sigma X[-1] \to X
\]

for any \( X \in \text{Syn}_E \) induced by the adjoint of the comparison map \( X(\Sigma P) \to \Omega X(P) \) for \( P \in \text{Sp}_E^{\text{fp}} \).

- We think of \( \tau \) as a parameter controlling the behaviour of the category \( \text{Syn}_E \). Heuristically:

\[
\begin{align*}
\text{Syn}_E^{\text{per}} & \xleftarrow{\text{invert } \tau} \text{Syn}_E & \xrightarrow{\text{kill } \tau} \text{Syn}_E^{\heartsuit} \\
\end{align*}
\]
More properties of $\text{Syn}_E$

\[ \text{Syn}_{E}^{\text{per}} \xleftarrow{\text{invert } \tau} \text{Syn}_E \xrightarrow{\text{kill } \tau} \text{Syn}_E^\heartsuit \]

- $\text{Syn}_{E}^{\text{per}} \simeq \text{Sp}_E$ as symmetric monoidal categories.
- $\text{Syn}_E^\heartsuit \simeq \text{Comod}_{E_* E}$ as symmetric monoidal categories.

We’ll also want:

- $\text{Syn}_E$ is \textit{complete}: every Postnikov tower converges.

\textbf{Remark}

This last property is not automatic. It holds when $E$ is Lubin-Tate theory using somewhat deep results related to vanishing lines in the $E$-based Adams SS.
Primer on synthetic spectra

Goerss-Hopkins obstruction theory

Application to Morava $E$-theory
Potential $n$-stages

Definition
A potential $n$-stage is an $\mathbb{E}_\infty$-algebra $R$ in $\text{Mod}_{1 \leq n}(\text{Syn}_E)$ such that $1_{\leq 0} \otimes 1_{\leq n} R$ is discrete.

- Denote the category of potential $n$-stages by $\mathcal{M}_n$.
- Extension of scalars along $1_{\leq n} \to 1_{\leq n-1}$ induce maps $\mathcal{M}_n \to \mathcal{M}_{n-1}$.
- Hence we get a tower

$$\mathcal{M}_\infty \to \cdots \to \mathcal{M}_n \to \mathcal{M}_{n-1} \to \cdots \to \mathcal{M}_0.$$  

- Under our completeness assumption, $\mathcal{M}_\infty \simeq \lim \mathcal{M}_n$. 
Potential $n$-stages

**Examples**

- A potential 0-stage is just an ordinary commutative algebra in $E_*E$-comodules.
- A potential $\infty$-stage is a $E$-local $E_\infty$-ring spectrum.
- Moreover, the map $\mathcal{M}_\infty \to \mathcal{M}_0$ sends $R$ to $E_*R$.
- So these potential stages interpolate between algebra and geometry.
We want to relate $\mathcal{M}od_1 \leq n$ with $\mathcal{M}od_1 \leq n-1$.

Note $1 \leq n$ is a square-zero extension of $1 \leq n-1$, so that there is a pullback square in $CAlg$ of the form

\[\begin{array}{ccc}
\mathbb{1}_{\leq n} & \to & \mathbb{1}_{\leq n-1} \\
\downarrow & & \downarrow i \\
\mathbb{1}_{\leq n-1} & \to & \mathbb{1}_{\leq n-1} \oplus \sum^{n+1} \pi_0 1[-n].
\end{array}\]
Proposition

There is a pullback square of categories

\[ \begin{array}{ccc}
\mathcal{M}od_{\leq n} & \xrightarrow{i_*} & \mathcal{M}od_{\leq n-1} \\
\downarrow & & \downarrow \quad i^* \\
\mathcal{M}od_{\leq n-1} & \xrightarrow{i_0} & \mathcal{M}od_{\leq n-1} \oplus \Sigma^{n+1} \pi_0 \mathbb{1} [-n].
\end{array} \]
Modules with sections

- Define a functor \( \Theta : \text{Mod}_{\mathbb{I} \leq n-1} \rightarrow \text{Mod}_{\mathbb{I} \leq n-1} \) by
  \[
  \Theta(M) = i_* i^*_0 M.
  \]

- The underlying spectrum of \( \Theta(M) \) is
  \[
  M \oplus \Sigma^{n+1}(M \otimes_{\mathbb{I} \leq n-1} \pi_0 \mathbb{1}[-n]).
  \]

- Let \( p : \mathbb{I} \leq n-1 \oplus \Sigma^{n+1} \pi_0 \mathbb{1}[-n] \rightarrow \mathbb{I} \leq n-1 \) be the projection. The unit of the adjunction \( p^* \dashv p_* \) induces a map
  \[
  \pi : \Theta M = i_* i^*_0 M \rightarrow i_* p_* p^* i^*_0 M \simeq \text{id}_* \text{id}^* M \simeq M.
  \]

Definition

\( \Theta \text{Sect}_{\mathbb{I} \leq n-1} \) is the category of \( \mathbb{I} \leq n-1 \)-modules equipped with a section \( s : M \rightarrow \Theta M \) of \( \pi \).
Modules with sections

Theorem
\( \mathcal{M}od_{\leq n} \simeq \Theta \mathcal{S}ect_{\leq n-1} \) as symmetric monoidal categories.

Proof.

- By the previous proposition,

\[
\mathcal{M}od_{\leq n} \simeq \{ (M, N \in \mathcal{M}od_{\leq n-1}, \alpha : i^* N \sim i_0^* M) \}.
\]

- Let \( \hat{\alpha} : N \to \Theta M \) be adjoint to \( \alpha \).

- Fact: \( \alpha : i^* N \to i_0^* M \) is an equivalence iff \( \pi \circ \hat{\alpha} \) is an equivalence (because \( \pi \circ \hat{\alpha} = p^* \alpha \) and \( p^* \) is conservative).
Modules with sections

Theorem
\( \mathcal{M}od_{\mathbb{1} \leq n} \simeq \Theta \text{Sect}_{\mathbb{1} \leq n-1} \) as symmetric monoidal categories.

Proof.

Therefore, we have

\[
\mathcal{M}od_{\mathbb{1} \leq n} \simeq \{(M, N, \alpha : i^* N \sim i_0^* M)\}
\]

\[
\simeq \{(M, N, \hat{\alpha} : N \to \Theta M) \mid \pi \circ \hat{\alpha} \text{ is } \simeq\}
\]

\[
\to \Theta \text{Sect}_{\mathbb{1} \leq n-1}
\]

sending \((M, N, \hat{\alpha})\) to \((M, \hat{\alpha} \circ (\pi \circ \hat{\alpha})^{-1}) \in \Theta \text{Sect}_{\mathbb{1} \leq n-1}\).

The fiber of this functor is identified with \(\{N \mid N \simeq M\}\), which is contractible.
Case when $R$ is a potential $(n - 1)$-stage

Lemma

Let $R$ be a potential $(n - 1)$-stage. Then the map $\pi : \Theta R \to R$ is a square-zero extension of $R$ by $\Sigma^{n+1} \pi_0 R[-n]$.

Proof.

▶ The fiber of the map

$$\pi : \Theta R \simeq R \oplus \Sigma^{n+1}(R \otimes \mathbb{1}_{n-1} \pi_0 \mathbb{1}[-n]) \to R$$

is just $\Sigma^{n+1}(R \otimes \mathbb{1}_{n-1} \pi_0 \mathbb{1}[-n])$.

▶ By definition of a potential $(n - 1)$-stage, this is concentrated in a single degree.

▶ Consequently, a lift of $R$ to a potential $n$-stage exists iff this extension admits a section by the previous theorem.
Cotangent complexes

- Square-zero extensions are controlled by cotangent complexes.
- More precisely:

\[
\{\text{square-zero extensions of } R \in \mathcal{C}A\text{lg}_A \text{ by } M \in \mathcal{M}od_R\} \\
\cong \pi_0 \text{Map}_{\mathcal{M}od_R}(\mathbb{L}_{R/A}, \Sigma M).
\]

- An extension is split iff the classifying map from the cotangent complex is null.
Obstructions to lifting objects

Theorem (Goerss-Hopkins)

Let $R$ be a potential $(n - 1)$-stage. There is an obstruction in the André-Quillen cohomology group

$$\text{Ext}^{n+2, n}_{\text{Mod}_{\pi_0 R}(\text{Syn}_E^\heartsuit)}(\mathbb{L}_{\pi_0 R/\pi_0 \mathbb{1}}, \pi_0 R)$$

which vanishes iff $R$ can be lifted to a potential $n$-stage.

Proof.

$\blacktriangleright$ $R$ lifts to a potential $n$-stage iff $\pi : \Theta R \to R$ admits a section
iff the map

$$\mathbb{L}_{R/\mathbb{1}_{\leq n-1}} \to \Sigma^{n+2} \pi_0 R[-n]$$

classifying the square-zero extension $\pi$ is null.
Obstructions to lifting objects

Proof.

Next, we describe the group of maps
\[ \mathbb{L}_{R/\mathbb{1} \leq n-1} \to \Sigma^{n+2} \pi_0 R[-n] \] in terms of algebra.

\[
\begin{align*}
\pi_0 \text{Map}_{\text{Mono}_R(Syn_E)}(\mathbb{L}_{R/\mathbb{1} \leq n-1}, \Sigma^{n+2} \pi_0 R[-n]) & \\
\simeq \pi_0 \text{Map}_{\text{Mono}_{\pi_0 R}(Syn_E)}(\pi_0 R \otimes_R \mathbb{L}_{R/\mathbb{1} \leq n-1}, \Sigma^{n+2} \pi_0 R[-n]) & \\
\simeq \pi_0 \text{Map}_{\text{Mono}_{\pi_0 R}(Syn_E)}(\pi_0 \mathbb{1} \otimes_{\mathbb{1} \leq n-1} \mathbb{L}_{R/\mathbb{1} \leq n-1}, \Sigma^{n+2} \pi_0 R[-n]) & \\
\simeq \pi_0 \text{Map}_{\text{Mono}_{\pi_0 R}(Syn_E)}(\mathbb{L}_{\pi_0 R/\pi_0 \mathbb{1}}, \Sigma^{n+2} \pi_0 R[-n]) & \\
=: \text{Ext}^{n+2, n}_{\text{Mono}_{\pi_0 R}(Syn_E)}(\mathbb{L}_{\pi_0 R/\pi_0 \mathbb{1}}, \pi_0 R).
\end{align*}
\]
Obstructions to lifting objects

Corollary

Let $A \in \mathcal{C}Alg(\mathcal{Comod}_{E^*E})$. There exists an inductively defined sequence of obstructions valued in André-Quillen cohomology groups $\text{Ext}_{\mathcal{Mod}_A(\mathcal{Comod}_{E^*E})}^{n+2,n}(\mathbb{L}A/E^*_*, A)$ for $n \geq 1$, which vanish iff there is an $\mathbb{E}_\infty$-ring spectrum $R$ such that $E^*_R \cong A$ as comodule algebras.
Obstructions to lifting maps

There’s a version of this machinery for lifting maps too, which is useful in determining the uniqueness of lifting objects.

Here is the setup:

- $R, S$ potential $n$-stages.
- $\phi : uR \to uS$ maps of corresponding potential $(n - 1)$-stages.

**Question**

*Does $\phi$ lift to a map $R \to S$?*

A similar argument as before shows...
Obstructions to lifting maps

Theorem (Goerss-Hopkins)

Let $R$ and $S$ be potential $n$-stages and $\phi : uR \to uS$ a map of corresponding potential $(n-1)$-stages. Then,

(a) There is an obstruction in $\text{Ext}^{n+1,n}_{\text{Mod}_{\pi_0 R}(\text{Syn}_E^{\heartsuit})}(\llbracket \pi_0 R/\pi_0 1 \rrbracket, \pi_0 S)$ which vanishes iff $\phi$ lifts to a map $R \to S$.

(b) In this case, the space of lifts of $\phi$ is

$$\text{Map}_{\text{Mod}_{\pi_0 R}(\text{Syn}_E^{\heartsuit})}(\llbracket \pi_0 R/\pi_0 1 \rrbracket, \Sigma^n \pi_0 S[-n]).$$

Corollary

Let $R$ and $S$ be $E$-local $\mathbb{F}_\infty$-rings, and let $A = E_* R$ and $B = E_* S$. Given a map $\phi : A \to B$ of commutative algebras in $E_* E$-comodules, there exists an inductively defined sequence of obstructions valued in $\text{Ext}^{n+1,n}_{\text{Mod}_A(\text{Comod}_{E_* E})}(\llbracket A/E_* \rrbracket, B)$ which vanishes iff there is an $\mathbb{F}_\infty$-ring map $\tilde{\phi} : R \to S$ such that $E_* \tilde{\phi} = \phi$. 
The mapping space spectral sequence

Corollary (Goerss-Hopkins)

Suppose we’re given a morphism $\phi : R \to S$ of $\text{Syn}^\text{per}_E$. There is a first quadrant spectral sequence converging conditionally to $\pi_{t-s}(\text{Map}_{\text{CAlg}(\text{Syn}_E)}(R, S))$ with $E_1$-page given by

$$E_1^{0,0} = \text{Map}_{\text{CAlg}(\text{Syn}_E)}(\pi_0 R, \pi_0 S),$$

$$E_1^{s,t} = \text{Ext}^{2s-t,s}_{\text{Mod}_{\pi_0 R}(\text{Syn}_E)}(\mathbb{I}_{\pi_0 R/\pi_0 \mathbb{I}}, \pi_0 S), \quad t \geq s > 0,$$

where $\pi_0 S$ is given the $\pi_0 R$-module structure via $\phi$.

Proof sketch.
This is the Bousfield-Kan spectral sequence applied to the tower $\{\text{Map}_{\mathcal{M}_s}(R_{\leq s}, S_{\leq s})\}$. 

\[\square\]
The mapping space spectral sequence

Corollary

Let $\phi : R \to S$ be a morphism of $E$-local $E_\infty$-rings. There is a first quadrant spectral sequence converging conditionally to

$$\pi_{t-s}(\text{Map}_{E_\infty}(R, S), \phi)$$

with

$$E_1^{0,0} = \text{Map}_{CAlg}(\text{Comod}_{E_*E})(E_*R, E_*S),$$

$$E_1^{s,t} = \text{Ext}_{\text{Mod}_{E_*R}(\text{Comod}_{E_*E})}^{2s-t,s}(\text{L}_E E_*R/E_*, E_*S), \quad t \geq s > 0,$$

where $E_*S$ is given the $E_*R$-module structure via $\phi$. 
Primer on synthetic spectra

Goerss-Hopkins obstruction theory

Application to Morava $E$-theory
The main theorem

Recall our main goal:

**Theorem (Goerss-Hopkins-Miller)**

Let $E$ be Lubin-Tate theory and $\mathcal{G}$ the Morava stabilizer group. Let $\mathcal{M}_E(E)$ be the moduli space of $E_\infty$-rings that are equivalent to $E$ as homotopy associative rings. Then, $\mathcal{M}_E(E) \cong B\mathcal{G}$.

**Proof.**

- Instead of realizing $E_*$ as an $E_\infty$-ring, we realize the comodule algebra $E_*E$, i.e., find an $E$-local $E_\infty$-ring $A$ such that $E_*A \cong E_*E$. This is enough by a universal coefficient spectral sequence argument.
Proving the main theorem

Proof.

- Apply Goerss-Hopkins obstruction theory: the obstructions to existence and uniqueness of lifts live in

\[ \text{Ext}^{s,t}_{\text{Mod}_{E*E}(\text{Comod}_{E*E})}(\mathbb{L}_{E*E/E*}, E*E). \]

We want to show these groups vanish unless \((s, t) = (0, 0)\).

- The free-forget adjunction from modules to comodules induces an isomorphism

\[ \text{Ext}^{*,*}_{\text{Mod}_{E*E}(\text{Comod}_{E*E})}(\mathbb{L}_{E*E/E*}, E*E) \cong \text{Ext}^{*,*}_{\text{Mod}_{E*}}(\mathbb{L}_{E*E/E*}, E*). \]
Proving the main theorem

Proof.

- Want to show $\text{Ext}^{*,*}_{\text{Mod}_{E_*}}(\mathbb{L}_{E_* E/E_*}, E_*) \cong 0$.

- Filter the target $E_*$ by powers of its maximal ideal $m = (p, u_1, \ldots, u_{h-1})$. This gives rise to a spectral sequence computing $\text{Ext}^{*,*}_{\text{Mod}_{E_*}}(\mathbb{L}_{E_* E/E_*}, E_*)$ with $E_2$-term
  $$\text{Ext}^{p,*}_{\text{Mod}_{E_*}/m}(\mathbb{L}_{E_* E/E_*} \otimes_{E_*} E_*/m, m^q/m^{q+1}).$$

- Suffice to show that $\mathbb{L}_{E_* E/E_*} \otimes_{E_*} E_*/m \simeq 0$. 


Proving the main theorem

Proof.

- Want to show \( \mathbb{L}_{E_*E/E_*} \otimes_{E_*} E_*/m \simeq 0 \).
- \( E_*E \) is flat over \( E_* \); by flat base change,

\[
\mathbb{L}_{E_*E/E_*} \otimes_{E_*} E_*/m \simeq \mathbb{L}(E_*E/m)/(E_*/m) \simeq E_* \otimes E_0 \mathbb{L}(E_0E/m)/(E_0/m).
\]

- \( E_0/m \simeq k \) is perfect, and so is \( E_0E/m \simeq \text{Hom}_{cts}(\mathbb{G}, k) \).

Claim

The cotangent complex of any morphism between perfect \( \mathbb{F}_p \)-algebras vanishes.

- So \( \mathbb{L}(E_0E/m)/(E_0/m) \simeq 0 \).
The cotangent complex of perfect $\mathbb{F}_p$-algebras

Claim

The cotangent complex of any morphism between perfect $\mathbb{F}_p$-algebras vanishes.

Proof.

The Frobenius automorphism induces an isomorphism on cotangent complexes, but the map is given by

$$dx \mapsto d(x^p) = px^{p-1} dx = 0.$$
Concluding the main theorem

Proof.

- Therefore, all obstructions to existence and uniqueness of an $\mathbb{E}_\infty$-ring structure on $E$ vanish.
- There is a unique $\mathbb{E}_\infty$-structure on $E$.
- Moreover,

$$\text{Aut}_{\mathbb{E}_\infty}(E) \cong \text{Aut}_{\mathcal{C}\text{Alg}}(\text{Comod}_{E^*E})(E^*E) \cong \mathbb{G}.$$ 

Remark

In general, $\text{Map}_{\mathbb{E}_\infty}(E(k_1,\Gamma_1), E(k_2,\Gamma_2))$ is homotopy discrete, with $\pi_0 = \text{Hom}_{\mathcal{FGL}}((k_1,\Gamma_1), (k_2,\Gamma_2))$. 
Thank you for listening!