1. INDUCTIVE FORMULATION SET-UP

Throughout, let \( X \) be a topological space. Let \( n \geq 2 \) and \( \zeta \to X \) a rank \( n \) vector bundle with an inner product. Consider the principal \( O(n) \)-bundle \( \tilde{X} \) of orthonormal frames of \( \zeta \).

**Question 1.** Show that \( X_0 = \tilde{X}/O(n-1) \) can be given the structure of a fiber bundle over \( X \) with fiber \( S^{n-1} \).

- How can you describe this bundle in terms of \( \zeta \)?
- Let \( p: X_0 \to X \) denote this \( S^{n-1} \)-bundle. Show that there is a universal rank \( n-1 \) vector bundle \( \zeta_0 \to X_0 \) with a map \( p: X_0 \to X \) so that
  \[
  p^* \zeta = \zeta_0 \oplus \mathbb{R}
  \]
- Describe \( \zeta_0 \) in terms of \( X_0 \) and \( \zeta \).
- Show that there is a functor
  \[
  \text{Bord}^{(X_0,\zeta_0)}_{n-1} \to \text{Bord}^{(X,\zeta)}_n
  \]
  What is this a functor of? \((\infty,n)\)-categories? \((\infty, n-1)\)-categories? How are you regarding both sides as that type of category?

You can read the top of page 53 of [1] for the answers to the above.

**Question 2.** Take \( X = BG \) and \( \zeta \) the rank \( n \) vector bundle \( \zeta_\xi = (\mathbb{R}^n \times EG)/G \) on \( BG \) associated to a continuous homomorphism \( \xi: G \to O(n) \). Describe \( X_0 \) (as defined in the previous problem) as a classifying space of some group.

See Remark 3.1.9 of [1] for the answer. More details about this pair \((BG, \zeta_\xi)\) can be found in Notation 2.4.21 of [1].

**Question 3.** Can you obtain \( \text{Bord}^{(X_0,\zeta_0)}_{n-1} \) from \( \text{Bord}^{(X,\zeta)}_n \) by removing the noninvertible \( n \)-morphisms? Why or why not?

See Remark 3.1.1 of [1] for the answer. The definition of a \((X,\zeta)\)-structure is Notation 2.4.16 of [1].

**Question 4.** Describe \( \Omega^k \text{Bord}^{(X_0,\zeta_0)}_n \) for \( k < n, k = n, k > n \).

**Question 5.** Recall that the \((n-1)\)-morphisms of \( \text{Bord}^{(X,\zeta)}_n \) are given intuitively by \((n-1)\)-manifolds with corners.

- Show that when \( X \) is a point, there is a unique \((n-1)\)-morphism in \( \text{Bord}^{(X,\zeta)}_n \) whose underlying manifold is \( S^{n-1} \).
• For $X$ this disjoint union of two points, how many such $(n-1)$-morphisms with underlying manifold $S^{n-1}$ are there?
• What about $X = S^{n-1}$ and $\zeta = TS^{n-1} \oplus \mathbb{R}$?
• For a general pair $(X, \zeta)$, give a description of the set of $(n-1)$-morphisms whose underlying manifold is $S^{n-1}$.
• Suppose $C$ is a symmetric monoidal $n$-category with duals and suppose you have an $n-1$ dimensional TFT $Z_0 : \text{Bord}^{(X, \zeta)}_{n-1} \rightarrow C$. Consider the functor $\Phi : X \rightarrow \Omega^{n-1}C$ sending $x \mapsto Z_0(S^{\zeta_x})$ where $S^{\zeta_x}$ is the fibre of the sphere bundle $\text{Sph}(\zeta) \rightarrow X$. Show that if we have an $n$-dimensional TFT $Z : \text{Bord}^{(X, \zeta)}_n \rightarrow C$ extending $Z_0$, then we have a family of 1-morphisms $\eta : 1 \rightarrow \Phi(x) \in \Omega^{n-1}C$ each exhibiting one half of the sphere as a right dual to the other.

Question 6. Read the statement of the inductive formulation, Theorem 3.1.8 of [1].

Question 7. Say $(X, \zeta) = (Bg, \zeta_\xi)$ as in Question 2. Reformulate the inductive formulation ([1, Thm. 3.1.8]) in terms of $G$ actions and $G$ equivariant maps.

See Remark 3.1.9 of [1] for an answer.

2. COBORDISM HYPOTHESIS FROM INDUCTIVE FORMULATION

Question 8. Assuming Theorems 2.4.6 and 2.4.18 in dimension $(n-1)$, what data is a symmetric monoidal functor

\[ Z_0 : \text{Bord}^{(X_0, \zeta_0)}_{n-1} \rightarrow C \]

equivalent to?

See (a1) on page 55 of [1] for an answer.

Question 9. Look over the Proof of Theorem 2.4.6 on page 55 of [1]. Give an outline of the proof of Theorem 2.4.6 (framed cobordism hypothesis) assuming Theorem 3.1.8 (inductive formulation).

REFERENCES