18.336J/6.335J :: Homework 3 :: Spring 2012 :: Due May 10

1. The Matlab function `solvepoissonmultigrid.m` implements a multigrid V-cycle for the 1D Poisson equation with Dirichlet boundary conditions.

   - (1pt) Which value of ω corresponds to the unweighted Jacobi iteration?
   - (2pts) In the example provided, $\omega = 1/4$ is a much better choice. Why is that?
   - (2pts) The residual smoothing command `res = [0; 1/4*res(I+1) + 1/2*res(I) + 1/4*res(I-1); 0];` is commented out in the code, because downsampling the residual directly is more accurate than downsampling it after smoothing. (You can try uncommenting the line and see the error go up.) How do you explain this behavior?
   - (Bonus) Performing a spline interpolation by changing `'linear'` to `'cubic'` or `'spline'` in the argument of `interp1` lowers the accuracy to a similar extent. How do you explain this behavior?

2. (5pts) In the fast multipole method, it is reasonable to use bilinear interpolation from values of the potential at the 4 vertices of a "target" square in order to obtain the potential everywhere inside the square. Explain how you would find equivalent (canonical) charges at the vertices of a "source" square from distributed sources inside the square, in a dual manner by using the *transpose* of bilinear interpolation. For well-separated boxes, show (either numerically or theoretically) that the accuracy of this operation is directly inherited from the accuracy of bilinear interpolation.