I. 1 Year Before Talbot, October?

one thing we do in math is tell a story of triumph, giving the clean, beautiful story. I'm going to pull back the curtain, giving a meandering story of ups & downs & ups

Location: MIT & then Münster ← My first conference & first time in Germany!

Event: Topological Modular Forms conference & the related pre-confnce. I was thinking about two questions, which I didn't see were so closely related:

"What happens at Talbot" = "space you can be vulnerable"

① Computing with tmf @ $p=3$:

I filled pages & pages w/ false starts, mistakes, & confusion!

For $p>3$, $tmf_{(p)}$ is essentially $BP\langle 2\rangle$, just as in Jeremy's talk. Work of Wilson, Adams, Margolis, ⇒ $\sim O(ku^2)$

For $p=2$, have an Adams SS, the E_2-term of which was extensively studied by Mahowald & Davis-Mahowald,

since $H^*(tmf;\mathbb{F}_2) = \mathcal{A} \otimes_{\mathcal{A}(2)} \mathbb{F}_2$, $\quad \mathcal{A}(2) = \langle Sq^1, Sq^2, Sq^4\rangle \subseteq \mathcal{A}$.

For $p=3$: had very little, but heuristically, should be "like ko at 2".

② Understanding Hopkins-Miller spectra.

What does this have to do w/ Talbot? <u>EVERYTHING</u> At MIT, we had an active group of <u>friends</u>. A lot of that is due to the careful community-minded work of Nora Ganter. At the conference, we met a group from ND. Nora worked to make sure we were all friends (and working in similar spaces). Back in the US, we were all energized. I had seen that "real mathematicians" had focused workshops. Nora pushed me to act on this idea, & she introduced us to a house MIT owned: Talbot House. Talbot was born!

... Sled Story

Picking up ②: Recall from Lior, Danny, Jeremy's talks:

A <u>formal group law</u> over R is a power series $x +_F y := F(x,y) \in R[[x,y]]$ satisfying

ⓐ $x +_F 0 = x$ ⓑ $x +_F y = y +_F x$ ⓒ $x +_F (y +_F z) = (x +_F y) +_F z$

A <u>homomorphism</u> of fgl is a power series $f(x) \in R[[x]]$ s.t. $f(x +_F y) = f(x) +_G f(y)$

These data are representable, as is "fgl + iso f w/ $f'(0)=1$"

(Quillen) MU_* carries the univ fgl & MU_*MU the univ strict iso

ⓑ+ⓒ ⇒ $\forall n \in \mathbb{N}$, $[n]_F(x) = \underbrace{x +_F \cdots +_F x}_{n} = nx \ (x^2)$ is a hom $F \to F$. <u>Prop</u> If $\mathrm{char}(R)=p$, then $[p]_F(x) = f(x^{p^n})$ some n.

In this case, $f(x) = v_n x + \ldots$ ← "detects" the self-maps Lior talked about

ht of F is at least n

Alg. geom talks about deformations of objects: - like in Maria's talk

$$\begin{tikzcd} \widetilde{F} \arrow[r] \arrow[d, squiggly, dash] & F \arrow[d, squiggly, dash] \\ A \arrow[r] & k \end{tikzcd} \qquad \begin{tikzcd} \widehat{G} \arrow[d] & \widehat{G} \arrow[l] \arrow[d] \\ \mathrm{Spec}\, A & \mathrm{Spec}\, k \arrow[l] \end{tikzcd}$$

nilpotent kernel

same space!

<u>Ex</u>: ·) $\mathbb{Z}/p^n \longrightarrow \mathbb{Z}/p$
·) $\mathbb{Z}[u_1,\ldots,u_k]/(p,u_1,\ldots,u_k)^m \longrightarrow \mathbb{Z}/p$

<u>Thm</u> (Lubin-Tate) There is a "universal deformation"

$$E(k,\Gamma)_0 \cong \underbrace{W(k)}_{\text{deform to char } 0}\underbrace{[[u_1,\ldots,u_{n-1}]]}_{\text{deform to hts } 1,\ldots,n-1}$$

This universal def inherits an action of $\mathrm{End}(\Gamma)$ & in particular of $\mathrm{Aut}(\Gamma)$ ← Danny & Lior & Jeremy

Thm (Goerss-Hopkins-Miller) There is an essentially unique functor $(k,F) \longmapsto E(k,\Gamma) \in \mathrm{Comm}$ w/

$\pi_*(E(k,F)) \cong E(k,F)_*$ ← $E(\mathbb{F}_{p^n}, F_{\text{Honda}}) =: E_n$ has an action of $\mathbb{G}_n = \mathrm{Gal}_{\mathbb{F}_p}(\mathbb{F}_{p^n}) \ltimes \mathrm{Aut}(\Gamma)$

(Devinatz-Hopkins) $L_{K(n)} S^0 \xrightarrow{\simeq} E_n^{h\mathbb{G}_n}$ ← Lior talked about this as a Gal extension

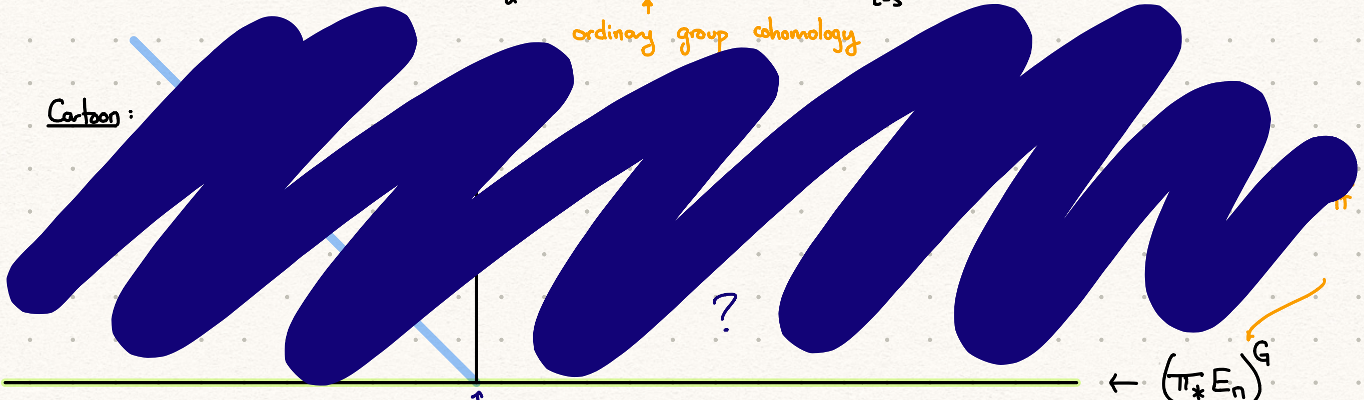
If $G \subseteq \mathbb{G}_n$ is finite, then $EO_n(G) := E_n^{hG}$. Have a Homotopy Fixed Point SS:

$$E_2^{s,t} = H^s(G; \pi_t E_n) \Rightarrow \pi_{t-s} EO_n(G).$$

ordinary group cohomology

$KU \Rightarrow KO$

Cartoon:



If $(|G|, p) = 1$, there is no higher cohomology!

Problem What is the action of G on $\pi_* E_n$?

This is where I was in '04 at the first Talbot. It was like how so many have described: I felt like part of a real community. CD, JNKF, AH & I decided this should be a recurring event

⇒ NSF grant. Our first attempt was a hot mess. But Chris Stark gave incredible, detailed feedback.

No booze, no pills...

First, not all G show up:

Thm (Hewitt) (p odd) $G \subseteq \mathrm{Aut}(F_{\text{Honda}}) \Rightarrow$ have a split exact sequence

$$\{1\} \to C_{p^r} \to G \to C_\ell \to \{1\}$$ where ① $p^{r-1}(p-1) \mid n$ & ② $(\ell, p) = 1$.

$\Rightarrow H^*(G; \pi_* E_n) \cong \left(H^*(C_{p^r}; \pi_* E_n)\right)^{C_\ell}$. Now $\mathrm{Aut}(\Gamma)$ has another description

$$\mathcal{O}_n = \underbrace{W(\mathbb{F}_{p^n})}_{\mathbb{Z}_{p^n}} \langle S \rangle \Big/ \begin{matrix} S^n = p \\ Sa = a^\varphi S \end{matrix}$$

Non-com. polys; lift of Frob. & $\mathrm{Aut}(\Gamma) \cong (\mathcal{O}_n)^\times$

Thm (Devinatz-Hopkins) ① There is a equivariant map

$$\begin{tikzcd} \mathcal{O}_n \arrow[r, dashed] \arrow[d] & \pi_{-2} E_n \arrow[d] \\ \mathcal{O}_n/p \arrow[r, "\cong"] & (\pi_{-2} E_n)/_{p, m^2} \end{tikzcd}$$

$\Rightarrow \mathrm{Sym}(\mathcal{O}_n)[\Delta^{-1}]^\wedge_I \xrightarrow{\cong} \pi_* E_n$

← k VS of dim n

② Up to associated graded, we can do this!

When $G = C_{p^r}$, $\mathcal{O}_n \cong \mathbb{W}(\mathbb{F}_{p^n})[\zeta_{p^r}]^{\oplus n/p^{r-1}(p-1)}$ w/ γ, a gen of C_{p^r}, acting as ζ_{p^r}-multiplication: very nice representation! $\mathrm{Ind}_{C_p}^{C_{p^r}}\left(\mathbb{Z}[C_p]/\mathbb{Z}\right)$

Cor There is a Leibnitz-Voltaire SS:

$$H^*\left(C_{p^r}; \mathrm{Sym}\left(\mathbb{Z}_p[\zeta_{p^r}]\right)^{\otimes m}\right) \Rightarrow H^*\left(C_{p^r}; \pi_* E_n\right).$$

I spent years banging my head against this! Shifted focus back to tmf-homology.
↑ No progress! Felt like a failure

III. Talbot 2007: time as a flat circle - tmf w/ Hopkins. Mike & I talked at length, and since my now husband was still in Boston, we made plans to talk over the summer. Doug Ravenel was visiting & ⇒

Theorem There is a G-equivariant lift

$$\begin{tikzcd}
\mathcal{O}_n \arrow[r, dashed] \arrow[d] & \pi_{-2} E_n \arrow[d] \\
\mathcal{O}_n/p \arrow[r] & \pi_{-2} E_n/_{p,\mathfrak{m}^2}
\end{tikzcd}$$

Cor $\pi_* E_n \cong \mathrm{Sym}(\mathcal{O}_n)\underbrace{[\Delta^{-1}]}_{}{}^{\wedge}_{I}$
understandable effect in $H^*(G; -)$.

This wasn't written up. The results here showed you can access the E_2-term of the ANSS for $E(k,F)^{hG}$.

Thm (Ravenel) For $p \geq 5$, β_{p^i/p^i} does not survive the ANSS for the sphere.

This is seen in the HFPSS for $E_{p-1}^{hC_p}$: All $\beta_{p^i/p^i} \mapsto$ non perm cycles! ⇝ Fails for $p=2,3$
error terms in same deg
$(p-1)=2$ ⇒ ht 2 theory

So want to look for higher height theories (& then bigger groups):

p	n	G
2	4	C_8
3	6	C_9

Danny gave a beautiful talk about what is tricky in the case $p=2$: How can we control the HFPSS? How do we prove it?

Like Mauricio described, this reframe question changed my research trajectory. We dodged the question, moving to "genuine" i.e. "complete" stable htpy (not Borel). I've been working here for a decade!

. . . And it was Talbot all along! In 2016, Doug & I acted as Talbot facilitators. And my math life was made only richer. I made so many new mathematical friends & now collaborators. The mathematics is fascinating, but what keeps me coming back are the friends I've made & the community that we have all built here together.