

SYLLABUS FOR 2013 TALBOT WORKSHOP

MENTORED BY MARK BEHRENS AND TYLER LAWSON

PREREQUISITES

Spectra. The stable homotopy category, homotopy groups of spectra, suspension spectra, Brown representability, smash products, ring spectra.

References: Schwede’s book on symmetric spectra [Sch], Adams’ Chicago Notes (part III) [Ada74], Appendix A of Ravenel’s Orange Book [Rav92], Margolis’ book, first couple of chapters [Mar83], available for download at:

www.math.rochester.edu/people/faculty/doug/otherpapers/margolis.djvu

Bousfield localization. Localization of spectra with respect to a cohomology theory. Localization at a prime, p -completion.

References: Tilman Bauer’s survey in the 2007 Talbot proceedings (Chapter 7: “The Hasse Square”):

<http://math.mit.edu/conferences/talbot/2007/tmfproc/>

Bousfield’s seminal and readable original paper [Bou79], Chapter 7 of Ravenel’s Orange Book [Rav92].

Complex cobordism, Formal groups, and BP. Definition and examples of formal groups, Quillen’s theorem relating MU_* to the Lazard ring, p -typical formal groups, BP .

References: The canonical reference is Part II of Adams’ Chicago notes [Ada74]. Appendix B of Ravenel’s Green book [Rav86], Appendix B of Ravenel’s Orange book [Rav92].

Adams and Adams-Novikov spectral sequences. Here the prerequisite is a more superficial knowledge of what an Adams spectral sequence is (or more precisely, the generalized Adams-Novikov spectral sequence associated to a cohomology theory E). We do not expect folks actually know how to compute with these things.

References: Section A.6 of Ravenel’s orange book [Rav92], part III of Adams’ Chicago notes [Ada74], Chapter 2 (and other parts) of Ravenel’s Green book [Rav86], convergence is nicely treated at the end of Bousfield’s paper [Bou79], Haynes Miller’s paper gives a unique homological perspective [Mil81a].

MONDAY: THE ADAMS-NOVIKOV SPECTRAL SEQUENCE, PERIODICITY, AND NILPOTENCE

Miller, Ravenel, and Wilson [MRW77], enacting a program of Morava, show that the algebra of formal group laws is linked to the structure of the stable homotopy category via the Adams-Novikov spectral sequence. This perspective led to an unprecedented computational understanding of the stable homotopy groups of spheres, and led Ravenel to propose a bold set of conjectures on how the structure of the stable homotopy category is controlled by the structure of formal groups

[Rav84]. With the exception of the telescope conjecture, all of these conjectures were famously proven by the combined efforts of Ethan Devinatz, Mike Hopkins, and Jeff Smith as the Nilpotence and Periodicity theorems [DHS88], [HS98].

(1) Introduction (Mark Behrens). An overview of the chromatic perspective on the stable homotopy category, from conceptual and computational perspectives. Summary of what we will be doing in this workshop.

(2) The Adams-Novikov spectral sequence and the Landweber exact functor theorem. The E_2 -term of the Adams-Novikov spectral sequence as for a flat ring spectrum E is given by

$$\mathrm{Ext}_{E_*E}(E_*, E_*)$$

for the Hopf algebroid (E_*, E_*E) . In the case where $E = MU$ or BP , the resulting groupoid has a nice formal group interpretation. The Landweber exact functor theorem uses the structure of this ring of cooperations to give a convenient criterion for producing complex-oriented cohomology theories starting with a formal group.

References: Ravenel's green book [Rav86] (esp. chapter 4), Appendix B of Ravenel's orange book [Rav92], Landweber [Lan76], Hovey-Strickland [HS05].

(3) The chromatic spectral sequence. The chromatic spectral sequence is an algebraic spectral sequence, arising from the structure of the BP -Hopf algebroid, which converges to the E_2 -term of the Adams-Novikov spectral sequence. Its n -line is v_n -periodic, and includes *Greek letter elements*. Goals: organize the Bockstein spectral sequences to clarify the Greek letter elements, and separate homotopy theory into layers by periodicity. Short exact sequences inducing long exact sequences on Ext_{BP_*BP} , and the associated chromatic spectral sequence converging to the Adams-Novikov E_2 -term for the sphere.

References: The original source is [MRW77], also see Ravenel [Rav86] (esp. chapter 5) and [Rav92] (esp. chapter 2).

(4) Nilpotence. Nishida's famous nilpotence theorem states that every positive degree element in the ring of stable homotopy groups of spheres is nilpotent. Devinatz-Hopkins-Smith proved a far-reaching generalization: any element the homotopy groups of any ring spectrum of positive Adams-Novikov filtration is nilpotent. Thus complex cobordism actually controls certain structural features of the stable homotopy category. Goals: statement of the nilpotence theorem, equivalent forms. Idea of the proof (time permitting): interpolate between the sphere and MU by the spectra $X(n)$, interpolate between the $X(n+1)$ and the $X(n)$, and use vanishing lines in the Adams-Novikov spectral sequence to show that each interpolating step detects nilpotence in the same way.

References: Nishida [Nis73], Ravenel's orange book [Rav92] (esp. chapter 9), Devinatz-Hopkins-Smith [DHS88].

(5) Periodicity. The nilpotence theorem is equivalent to a structural theorem on the stable homotopy category: the *thick subcategory theorem*. This theorem allows one to realize that the periodicity phenomena algebraically present in the chromatic spectral sequence can actually be realized topologically. Motivation: Self-maps of finite complexes inducing 8-periodic and $2p-2$ -periodic families in stable homotopy groups of spheres, and their detection using K -theory. v_n -self maps, equivalent

formulations. Detecting them using BP -homology. Idea of proof that the category of spectra equipped with a v_n -self map is a thick subcategory.

References: Ravenel's orange book [Rav92] (esp chapter 6), Devinatz-Hopkins-Smith [DHS88].

TUESDAY: LOCAL THEORY

The global perspective outlined in Monday's talks suggest that stable homotopy theory is in some sense governed by a sequence of v_n -periodic layers. Tuesday's talks are focused on how to understand this perspective more precisely, and explain Morava's insight into the computability of these layers.

(6) Morava E -theory. Morava's *change of rings isomorphism* states that the E_1 -term of the chromatic spectral sequence can be computed as the cohomology of the *Morava stabilizer group* (the group of automorphisms of the Honda formal group) with coefficients in the Lubin-Tate deformation ring. There are cohomology theories which geometrically realize the Honda height n formal group (Morava K -theory $K(n)$) and its Lubin-Tate universal deformation (Morava E -theory E_n).

References: Ravenel's green book [Rav86] (esp chapter 6) and orange book [Rav92] (esp chapter 4), Miller and Ravenel [MR77], Devinatz [Dev95], Morava [Mor85].

(7) The chromatic tower. The *chromatic tower* is a tower of localizations with respect to the Johnson-Wilson theories $E(n)$, which topologically realizes the chromatic spectral sequence. The tower can be inductively understood in terms of monochromatic layers, and in terms of $K(n)$ -localizations. The monochromatic layers are related to the generalized Moore spectra arising from the periodicity theorem. The Hopkins-Ravenel *chromatic convergence theorem* gives criteria for the convergence of the chromatic tower.

References: The introductions of [GHMR05] and [BL10] provide nice overviews. Ravenel [Rav86] (esp chapter 5) and [Rav92] (esp chapter 8). Many important proofs can also be culled from [Hov95]. The proof of the chromatic convergence theorem is in [HR92].

(8) The $K(n)$ -local sphere. The chromatic tower focuses attention on the study of $K(n)$ -local spectra, and in particular the $K(n)$ -local sphere. While the Landweber exact functor theorem implies the existence of the Morava E -theories E_n , it only implies that the Morava stabilizer group acts on E_n in the homotopy category. The Goerss-Hopkins-Miller theorem shows that this action can be refined essentially uniquely to a point-set level action. Devinatz and Hopkins used this to give a topological formulation of the Morava change of rings theorem: $E_n^{hG_n} = S_{K(n)}$. Here, the homotopy fixed points are a kind of continuous homotopy fixed points, as the Morava stabilizer group G_n is profinite. More generally, Devinatz-Hopkins defined E_n^{hH} for all closed subgroups $H \leq G_n$.

References: Devinatz-Hopkins [DH04], Rezk [Rez98], the Goerss-Hopkins E_∞ refinement [GH04], another perspective on continuous homotopy fixed points [BD10].

(9) The J spectrum. This lecture works through the details of the general theory presented throughout the day in the historically important case of $n = 1$. In this case, Morava E -theory is p -complete K -theory, and the action of the Morava

stabilizer group is nothing but the action of stable (prime-to- p) Adams operations. Adams & Baird showed that in this case the $K(1)$ -local sphere is the classical J -spectrum, whose homotopy groups are essentially the image of the J -homomorphism.

References: Ravenel [Rav84], Bousfield [Bou79], Adams' $J(X)$ papers, especially $J(X)$ IV [Ada66].

WEDNESDAY: LOCAL AND GLOBAL v_n -PERIODIC COHOMOLOGY THEORIES

Historically, the first manifestation of periodicity in algebraic topology was in the form of Bott periodicity in K -theory. Real K -theory, the fixed points of complex K -theory under the conjugation action, is a more sensitive homotopy invariant at the prime 2. Hopkins and Miller observed that similarly more powerful cohomology theories could be obtained from the Morava E_n -theories by taking homotopy fixed points by finite subgroups: these give the cohomology theories EO_n . At the primes 2 and 3, they saw that EO_2 had connections to elliptic curves, and this led to the global v_2 -periodic theory of topological modular forms. Topological automorphic forms are a natural v_n -periodic analog.

(10) The higher real K -theories EO_n . As seen on Tuesday, the Goerss-Hopkins-Miller theory allows one to take homotopy fixed points of E_n with respect to any closed subgroup of G_n . When the the subgroup is taken to be a maximal finite subgroup, the resulting homotopy fixed point spectrum is called EO_n , and is regarded as a “higher real K -theory spectrum” (the case of $p = 2$ and $n = 1$ is 2-complete real K -theory). The case of $n = p - 1$ is especially well-studied and computable: Gorbounov-Hopkins-Mahowald observed that maximal finite subgroup is the automorphism group of an Artin-Schrier curve, and the associated formal group is a summand of the formal completion of the resulting Jacobian. Nave showed that computational properties of the theories EO_{p-1} give strong non-existence results for Smith-Toda complexes. The case of $n = 2$ is a local version of TMF for $p = 2, 3$.

References: Nave [Nav10], Hopkins-Mahowald [HM], Gorbounov-Mahowald [GM00], Behrens-Hopkins [BH11].

(11) Topological modular forms. A complex orientable cohomology theory whose formal group is the formal group of an elliptic curve is called an *elliptic cohomology theory* (historically the first was due to Landweber-Ravenel-Stong). The universal such theory (the theory associated to the univesal elliptic curve over the moduli *stack* of elliptic curves) is the theory of *Topological Modular Forms* (TMF). $K(2)$ -locally, TMF is a product of homotopy fixed points of E_2 by finite subgroups of the Morava stabilizer group. As the moduli stack of elliptic curves has isotropy of order 2 and 3, TMF has elaborate 2 and 3 torsion which detect many v_2 -periodic families at these primes in its Hurewicz image. The homotopy groups of TMF are periodic of order 576.

References: Talbot 2007 proceedings:

<http://math.mit.edu/conferences/talbot/2007/tmfproc/>,

Hopkins' ICM addresses [Hop02], [HM], Landweber-Ravenel-Stong [LRS95], Lurie's survey [Lur09], [Bau08].

(12) Topological automorphic forms. In the previous lecture we introduced the theory of topological modular forms, which was a kind of global v_2 -periodic cohomology theory (in contrast to the local EO_2), which arose out of studying moduli spaces of algebro-geometric objects with associated 1-dimensional formal groups of height up to height 2. Similar moduli spaces of algebro-geometric objects (abelian varieties with complex multiplication) which give rise to 1-dimensional formal groups of heights up to height n were introduced by Shimura (Shimura stacks of type $U(1, n-1)$). Lurie proved a generalization of the Goerss-Hopkins-Miller theorem, which replaces the role of formal groups with p -divisible groups. When this is applied to the Shimura stacks mentioned above, one obtains global v_n -periodic cohomology theories of *topological automorphic forms*.

References: Tyler's survey [Law09], Goerss' survey [Goe09], the memoir [BL10].

THURSDAY: THE $K(n)$ -LOCAL STABLE HOMOTOPY CATEGORY

The $K(n)$ -localizations of the stable homotopy category have special and unusual properties that are quite distinct from their unlocalized counterpart. Tate spectra, a measurement of the difference of finite homotopy limits and colimits, vanish, in turn connecting the different $K(n)$ -local categories in mysterious ways. Eilenberg-MacLane spaces are non-trivial only in finitely many degrees - giving finite postnikov systems unusual $K(n)$ -local properties. The collection of invertible objects (the Picard group) is p -adically continuous instead of discrete, giving a continuous indexing on the $K(n)$ -local homotopy groups. These local homotopy groups exhibit a special duality (Gross-Hopkins duality) not seen on the global level. Furthermore, the E_n -cohomology of finite groups is determined by a character theory generalizing the classical character theory describing the K -theory of BG via the Atiyah-Segal completion theorem.

(13) Tate spectra. Tate spectra are defined by a norm cofiber sequence

$$X_{hG} \xrightarrow{\text{norm}} X^{hG} \rightarrow X^{tG}$$

Greenlees-Sadofsky, Kuhn, show that when X is $K(n)$ -local, the Tate spectrum is $K(n)$ acyclic. This has several consequences (1) it implies that the $K(n)$ -locally the classifying spaces of finite groups are self-dual, and (2) that in general the Tate spectrum must be supported on a lower chromatic level. This "Blue shift" phenomenon shows the Tate spectrum somehow connects the chromatic levels. The case of G being cyclic of order p is especially nice, when X is complex orientable, and the Tate spectrum is seen to take Morava E_n -theories to v_{n-1} -periodic theories.

References: Greenlees-May's memoir [GM95], Ando-Morava-Sadofsky [AMS98], Greenlees-Sadofsky [GS96], Kuhn [Kuh04], Mahowald and Ravenel discuss consequence for the "root invariant" [MR93].

(14) Morava K -theory of Eilenberg-Mac Lane spaces. Eilenberg-MacLane spectra exhibit surprising finiteness properties $K(n)$ -locally. Computation of the Morava K -theory of $K(\mathbf{Z}, 2)$, $K(\mathbf{Z}/p, 1)$ and their relation to the formal group law. Classifying spaces and the Rothenberg-Steenrod spectral sequence. Ravenel-Wilson's inductive method for calculating differentials in this spectral sequence. $K(n)$ -acyclicity of Eilenberg-MacLane spectra.

References: Ravenel-Wilson [RW80], Eric Peterson discusses a nice perspective in arXiv:1109.5989.

(15) Gross-Hopkins duality and Pic . The picard group of a symmetric monoidal category is the collection of invertible objects. The only invertible objects in the stable homotopy category are the spheres. The picard groups of the $K(n)$ -local categories are p -adically continuous, and contain many exotic elements. Gross and Hopkins show that homotopy groups of $K(n)$ -local spectra exhibit a duality when indexed on the picard group, and the dualizing object is a very important and potentially exotic element of the picard group.

References: Strickland [Str00], Devinatz [Dev97], Gross-Hopkins [HG94], Hopkins-Mahowald-Sadofsky [HMS94], the v_2 -periodic case is discussed in [Beh12].

(16) Hopkins-Kuhn-Ravenel character theory. The Atiyah-Segal completion theorem implies that the K -theory of BG is a completion of the representation ring of G , which in turn injects into the ring of class functions on G . Hopkins-Kuhn-Ravenel showed that the Morava E_n -theory of BG injects into the ring of functions on conjugacy classes of n commuting elements of G .

References: Hopkins-Kuhn-Ravenel [HKR00].

FRIDAY: VISTAS

In this final day we explore some more recent or ongoing topics, with an eye to future research directions.

(17) The telescope conjecture and chromatic splitting conjecture. The last remaining conjecture of Ravenel's original conjectures is the telescope conjecture - it conjectures that two different kinds of localization are equal (finite localization and localization) with respect to $E(n)$. The localization with respect to $E(n)$ is what is relevant for the chromatic tower, but finite localization is more closely tied to the classical notion of v_n -periodicity. It is true for $n = 1$ (see [Bou79]) by explicit computation (Miller for $p > 2$, Mahowald for $p = 2$). Explicit computation seems to show that the telescope conjecture is false for $n = 2$, but decades of work have proved fruitless in nailing this down. Hopkins has proposed another conjecture, which is a kind of Hasse principle, called the chromatic splitting conjecture: that the homotopy groups of spheres injects into the product of its $K(n)$ -localizations.

References: Ravenel [Rav84], Miller [Mil92], Miller [Mil81b], Mahowald-Ravenel-Shick [MRS01], Hovey's treatment of chromatic splitting [Hov95].

(18) Resolutions of the $K(2)$ -local sphere. The Adams-Baird result on Tuesday presented a short resolution

$$KO_p \rightarrow KO_p$$

of the $K(1)$ -local sphere. Goerss, Henn, Mahowald, and Rezk generalized this picture to the $K(2)$ -local sphere for the prime 3, showing there was an explicit finite resolution of it by homotopy fixed point spectra of E_2 by finite subgroups of the Morava stabilizer group. Half of this resolution has a TMF -description via moduli of elliptic curves and the building for GL_2 . Henn has extended the Goerss-Henn-Mahowald-Rezk work to handle other primes, and other chromatic levels.

References: Goerss-Henn-Mahowald-Rezk [GHMR05], Henn's paper [Hen07], Mark paper on buildings [Beh07], Mark and Tyler's density result [BL06]

References: Bousfield [Bou94], Kuhn [Kuh89], [Kuh08], [Kuh07], [Mah82], [Tho90], a nice summary of the past two papers is in section 1.5 of [Rav86], [Rez06].

(19) Other future directions (Tyler). Tyler will discuss a selection of other current and future research directions. For instance, Ausoni, Rognes, and others are pursuing a program of *red shift*, where K -theory increases chromatic filtration, Tyler and his collaborators have been able to establish some elusive E_∞ -structures on $BP\langle n \rangle$ by modeling these spectra with topological automorphic forms, etc...

CUT TALKS

The β family and the f invariant. Miller-Ravenel-Wilson described the 2-line of the Adams-Novikov spectral sequence, showing it was generated by the Greek letter elements $\beta_{i/j,k}$ for suitable indices (i, j, k) . The beta-family is a v_2 -periodic analog of the image of J . Adams used K -theory to construct his e invariant, which lands in \mathbb{Q}/\mathbb{Z} , and detects the elements of the image of J , and showed that the image of J is controlled by divisibility properties of denominators of Bernoulli numbers. Laures used TMF to construct an f -invariant of the 2-line of the ANSS, which lands in a quotient of Katz's ring of divided congruences, and which detects the beta family. Mark used the building resolution and his and Tyler's density result to give a bijective correspondence to elements of the beta family and certain explicit congruences between modular forms — this turns out to be a characterization of the f invariants of beta elements.

References: Miller-Ravenel-Wilson [MRW75], Laures' f invariant [Lau99] Mark's paper on beta families [Beh09], joint paper relating the two [BL09]

The Bousfield-Kuhn functor and unstable v_n -periodic homotopy. Up to this point everything we have talked about is stable, but there is also an unstable version of v_n -periodic homotopy. Mahowald (at the prime 2) and Thompson ($p > 2$) computed unstable v_1 -periodic homotopy groups of spheres by showing that unstable v_1 -periodic homotopy is isomorphic to metastable v_1 -periodic homotopy (and computed the latter using stable methods). Bousfield and Kuhn observed that these unstable v_n -periodic homotopy of X as the stable homotopy of a spectrum $\Phi_n(X)$ — moreover, the functor Φ_n factors $K(n)$ -localization through the zeroth space of the spectrum. This displays yet another bizarre property of $K(n)$ -localization, and leads to many surprising results, including Rezk's logarithm. Applied to the Goodwillie tower of the identity, Arone and Mahowald showed the Bousfield-Kuhn functor gives a vast generalization of the original Mahowald-Thompson program.

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