de Finetti Theorems for Markov Chains: Some References and Three Applications

1) Usual de Finetti theorems for exchangeable sequences are masterfully developed in O. Kallenberg's book: 'Probabilistic Symmetries and Invariance Principles' (2009). For generalizations to partial exchangeability (including Markov chains, graph limits and much else) see Diaconis, P. and Freedman, D. 'Partial Exchangeability and Sufficiency' (easiest to find on my home page, papers (1984)). To understand why (some) statisticians care and what de Finetti thought he was doing, see Diaconis, P. and Skyrms, B. (2017) 'Ten Great Ideas about Chance'.

2) de Finetti's theorem for Markov chains was proved in Freedman's thesis: Freedman, D. (1962) 'Mixtures of Markov Processes', Ann. Math. Statist. for stationary processes. This is sharpened in Diaconis, P. and Freedman, D. (1980) 'de Finetti's Theorem for Markov Chains.' Freedman wrote several later papers (more general state spaces, continuous time) but this is still open.

3) Random walk with reinforcement was introduced in Diaconis, P. (1988) 'Recent Progress in de Finetti's Notions of Exchangeability' (easiest to find on my home page). Robin Pemantle's many nice theorems are surveyed in his 'A Survey of Random Processes with Reinforcement' Probab. Surveys (2007). For the latest, see Sabot, C. and Tarres, P. (2015) 'Edge Reinforced Random Walk, Vertex Reinforced ~~Random~~ Jump Processes and the Supersymmetric Hyperbolic Sigma Model'. We

4) The application to DNA sequencing (and finite versions with rates) is in Zaman, A. (1984) 'Urn Models for Markov Exchangeability' and 'A Finite Form of de Finetti's Theorem for Stationary Markov Chains' (1986). See also Kandel, D., Matias, Y., Unger, R. and Winkler, P. 'Shuffling Biological Sequences' (1996).

5) For applications in Bayesian statistics (and protein folding) see Diaconis, P. and Rolles, S. (2006) 'Bayesian Analysis for Reversible Markov Chains', Bacallado, S., Chodera, J. and Pande, V.J. (2010) 'Bayesian Comparison of Markov Models of Molecular Dynamics' and Bacallado, S. (2011) 'Bayesian Analysis of Variable Order Reversible Markov Chains'

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Let $G=(V,E)$ be a finite undirected graph (loops allowed). Let $\Delta = \{x=(x_e)_{e\in E},\ x_e>0,\ \sum_{e\in E} x_e = 1\}$. Set $\ell = |V| + |E_{loop}|$, $m = |E|$. Choose $C_1, C_2 \cdots C_{m-\ell+1}$ cycles in G, an additive basis for H_1. Define (for $x\in\Delta$) $A(x)_{ij}$, $1\le i,j\le m-\ell+1$,

$$A_{ii}(x) = \sum_{e\in C_i} \frac{1}{x_e} \qquad A_{ij}(x) = \sum_{e\in C_i\cap C_j} \pm\frac{1}{x_e}, \quad i\neq j.$$

For initial weights $a_e\in(0,\infty)^E$ define

$$\pi_{v_0,a}(x) = Z^{-1}\, \frac{\prod_{e\in E\setminus E_{loop}} x_e^{a_e-\frac{1}{2}} \prod_{e\in E_{loop}} x_e^{(a_e/2)-1}}{x_{v_0}^{a_{v_0}/2} \prod_{v\in V\setminus v_0} x_v^{(a_v+1)/2}} \sqrt{\det(A(x))}$$

$$Z = \Big\{\prod_{e\in E}\Gamma(a_e) \Big/ \Gamma(a_{v_0}/2) \prod_{v\in V\setminus v_0}\Gamma((a_v+1)/2) \prod_{e\in E_{loop}}\Gamma\Big(\frac{a_e+1}{2}\Big)\Big\}\ \frac{(m-1)!\ \pi^{(\ell-1)/2}}{2^{1-\ell+\sum_{e\in E} a_e}}.$$

This is the limiting density of edge reinforced random walk starting at v_0 with initial wts a_e.