Quantum groups and 2-representations

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UCLA

Representation Theory Days

in honor of George Lusztig

MIT

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$$H_n = k\langle X_1, \ldots, X_n, T_1, \ldots, T_{n-1} \mid X_i X_j = X_j X_i,\ T_i^2 = 0,\ T_i T_j = T_j T_i\ (|i-j|>1),$$
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General g : $E_i^n \simeq [n]_q!\, E^{(n)}$

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- Non-simply laced case: basis $U_q(\mathfrak{g})^{>0}$, structure constants in $\mathbb{Z}_{\geq 0}[q, q^{-1}]$

$\neq$ Canonical basis. What is that basis?

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(Conjectural) • 4d topology (Crane–Frenkel program)

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Example $D^b_c(\mathrm{Grass}(\mathbb{C}^n)) \simeq D^b_c(\cdot \amalg \cdot)^{\otimes n}$ as 2-rep of $\mathfrak{sl}_2$
(B-smooth)

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$A \circledast A' = T_{A \otimes A'}(M)/(I)$, $A' \circledast A = T_{A' \otimes A}(\Gamma M^\vee)/(I^\perp)$ (Koszul duality)

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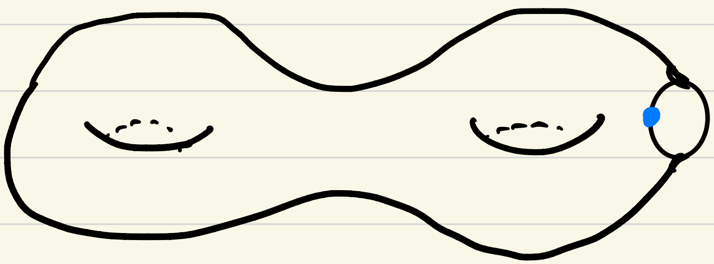
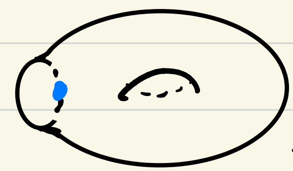
HF() $\simeq$ HF() $\otimes$ HF()

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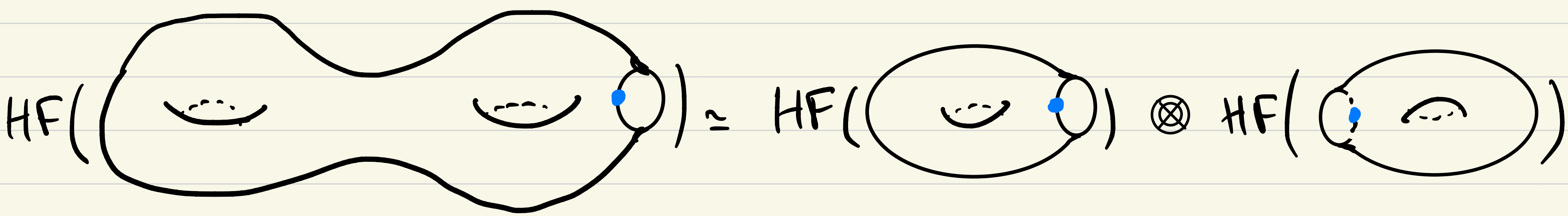
(joint with A. Manion)

Heegaard Floer: 2,3,4d TQFT (Oszváth-Szabó, Lipshitz-Oszváth-Thurston)

2d $\Sigma \mapsto HF(\Sigma) = \mathrm{Fuk}(S^g\Sigma)$ (using Auroux)

2-rep of $\mathfrak{gl}(1|1)^{>0}$ on $HF(\Sigma)$ for $\bigcirc \hookrightarrow \partial\Sigma$

$\mathcal{U}(\mathfrak{gl}(1|1))^{>0}$: $1, E, E^2=0$



Conj | Link invariant for vector 2-rep = $\widehat{HFK}$ (Ozsváth-Szabó, Rasmussen)