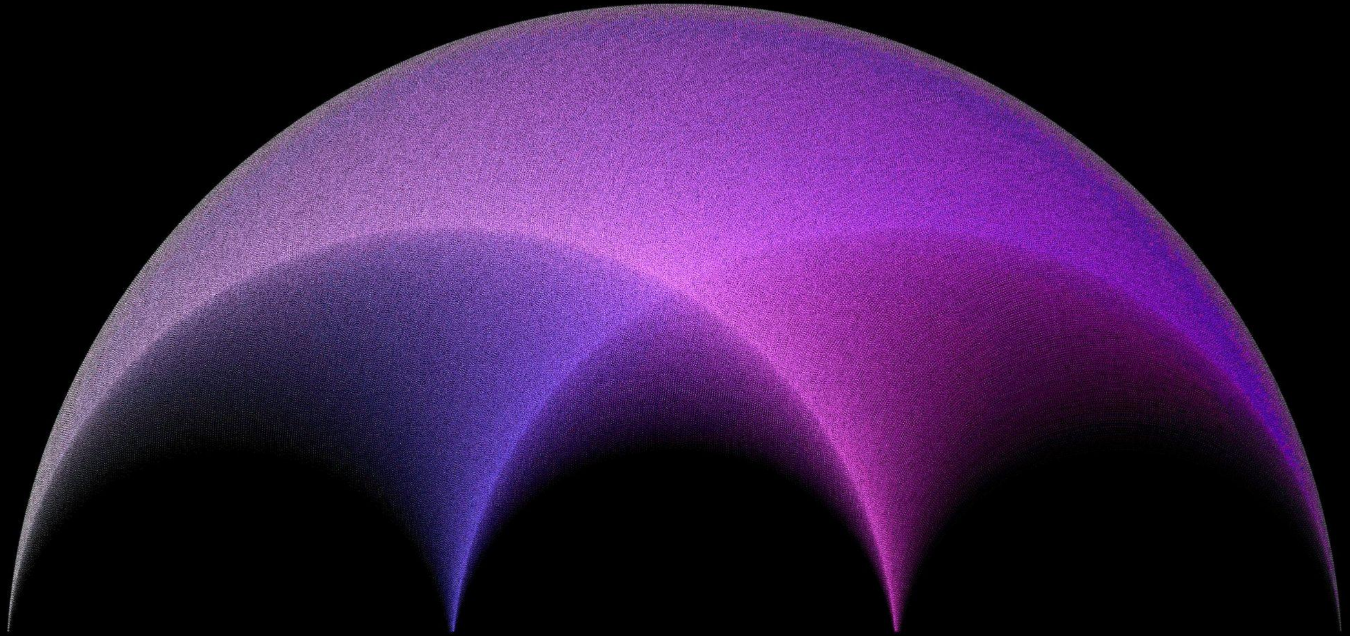
Girls and Math

C. Kenneth Fan, ‘95
Girls’ Angle

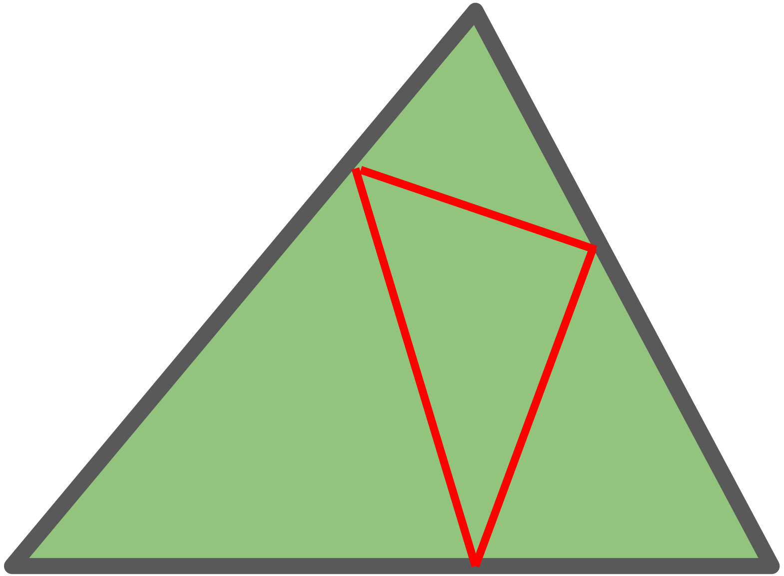


Representation Theory Days
In Honor of George Lusztig

Theorem: Girls are fabulous mathematicians!

Theorem: Girls are fabulous mathematicians!

Proof: This will follow from a series of propositions.



Definition: A light path in a polygon is a laser beam that bounces consecutively on the sides before closing up and forming a loop.

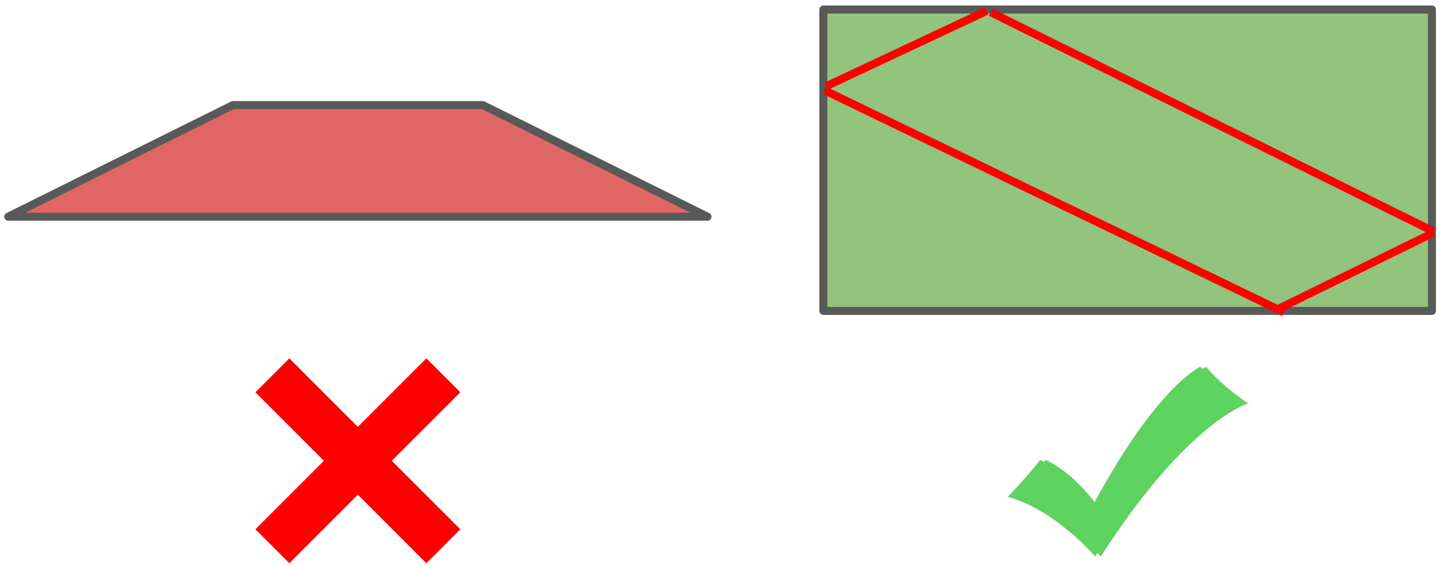
It's been known since the 18th century that a triangle has a unique light path if and only if it is acute (Giovanni Fagnano).

? ?

Proposition 1. A quadrilateral has a light path if and only if

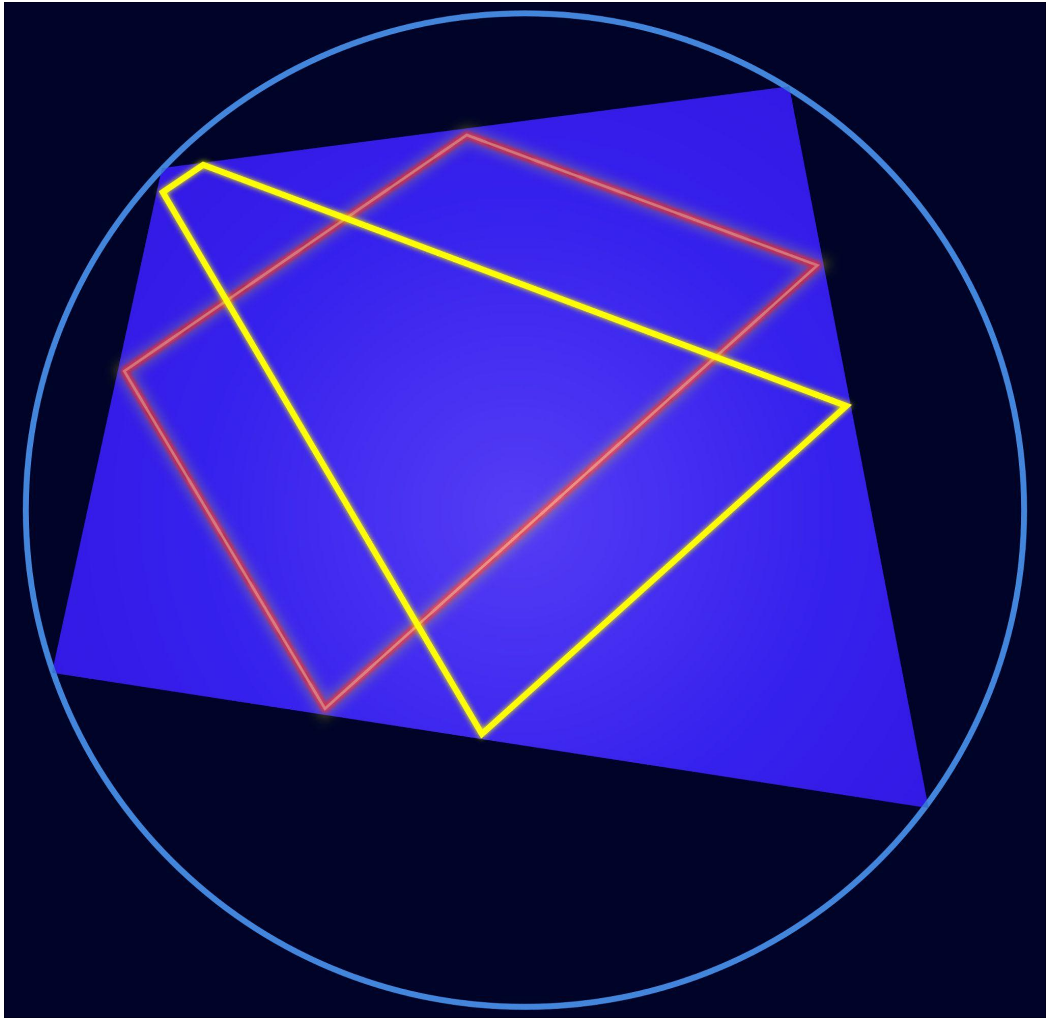
____?



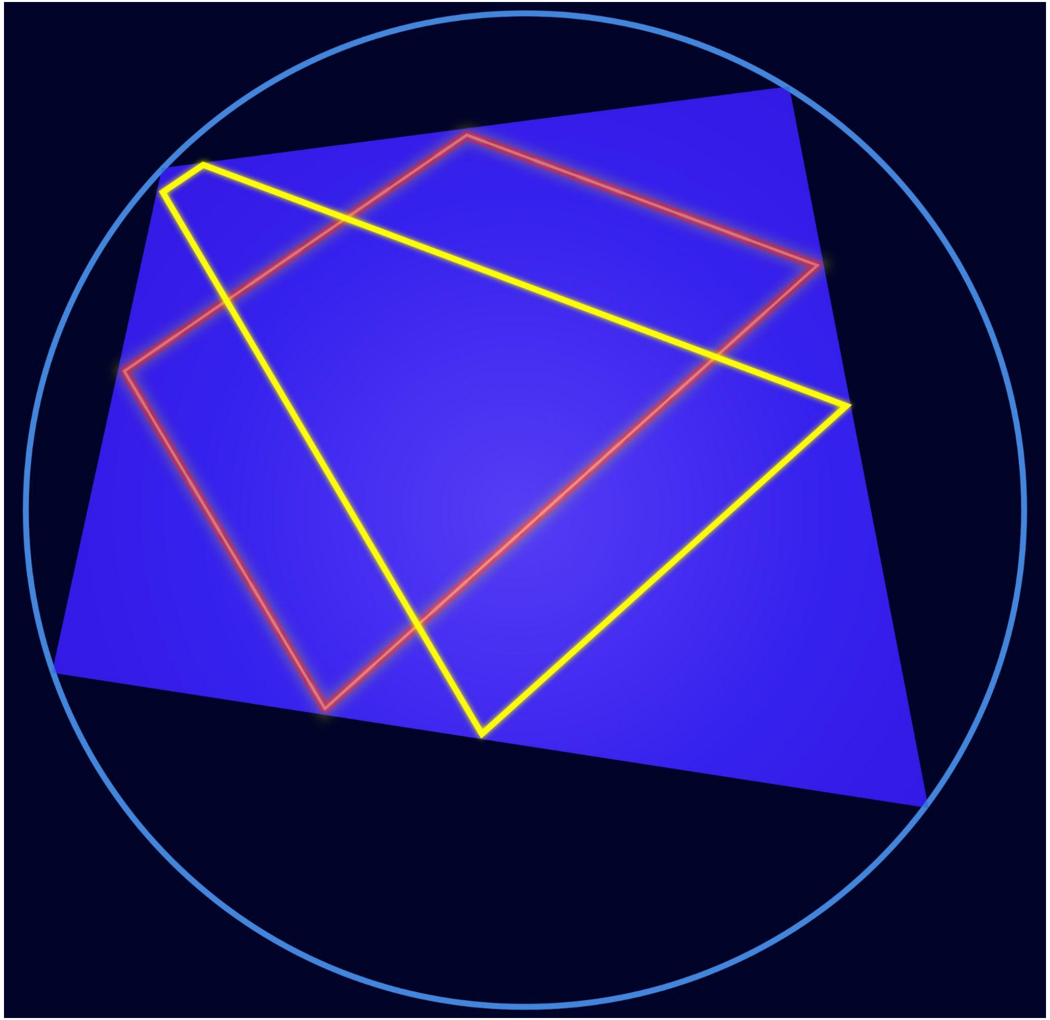
Proposition 1. A quadrilateral has a light path if and only if **it is cyclic and contains the center of its circumcircle**.

- Katherine Knox (12 years old)

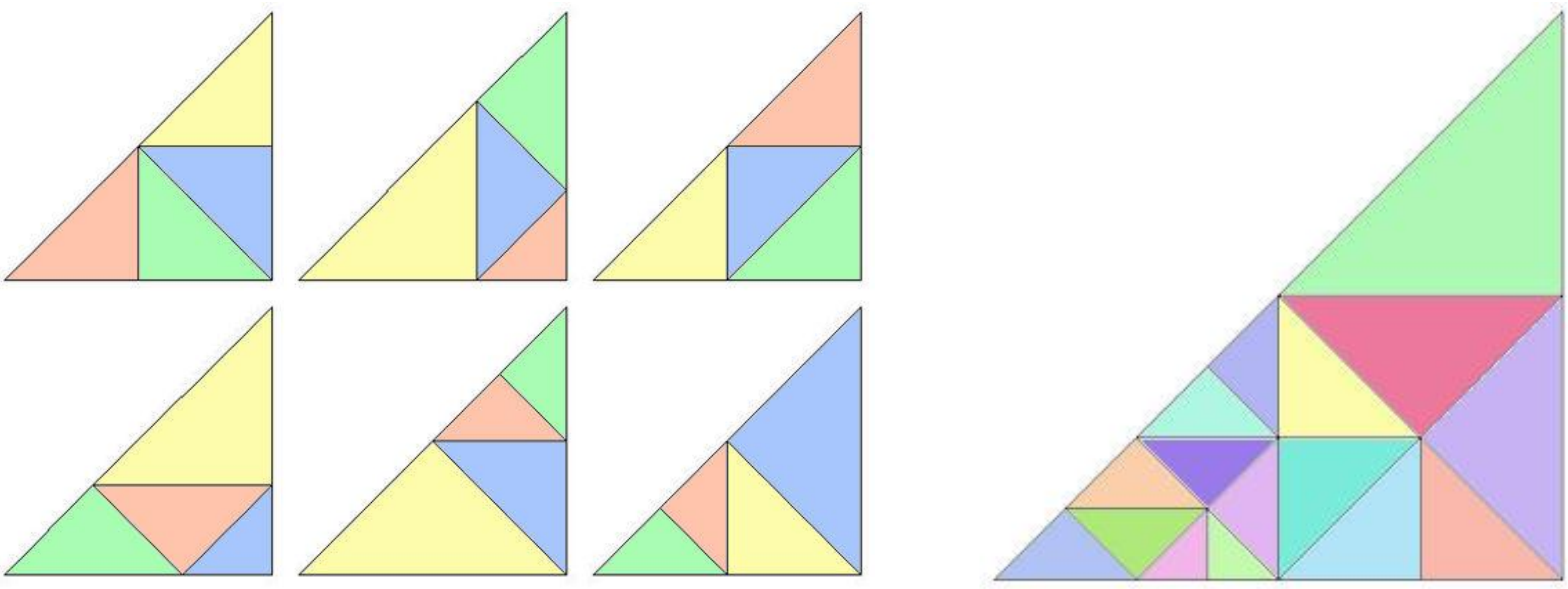
K. Knox, Billiard Circuits in Quadrilaterals, *The American Mathematical Monthly*, 130(8), 755-759 (2023).



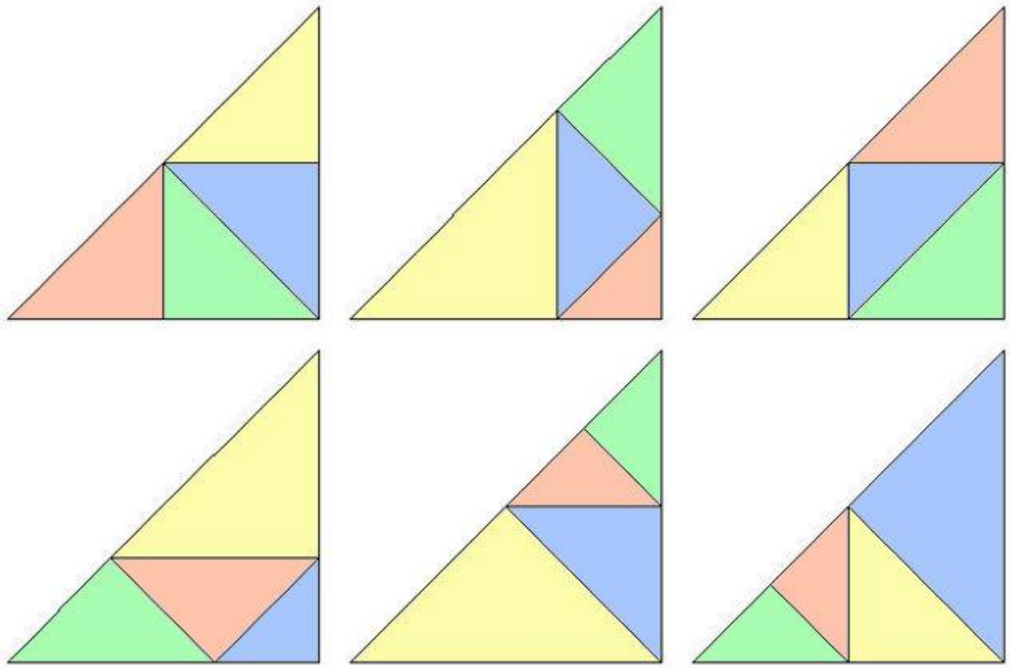
K. Knox, Billiard Circuits in Quadrilaterals, *The American Mathematical Monthly*, 130(8), 755-759 (2023).

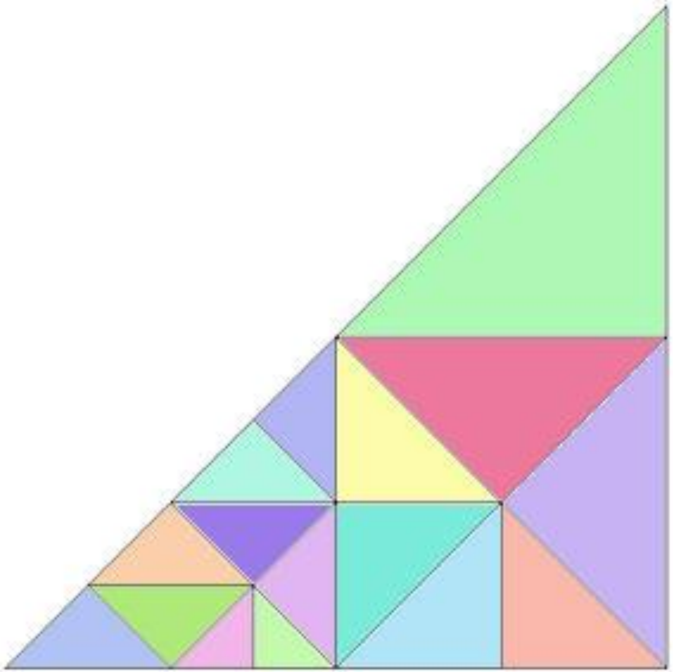


Recently, it came to my attention that this result was also found using a different method by Don Chakerian in 1963. See: On Quadrilaterals of Minimal Perimeter Inscribed in a given Quadrilateral, *American Mathematical Monthly*, 70(3), 294-298 (1963).



Proposition 2. Let $f(n)$ be the number of edge-to-edge n-tilings of a right isosceles triangle with right isosceles triangles. Then $f(n)$ is bounded by ______ ?



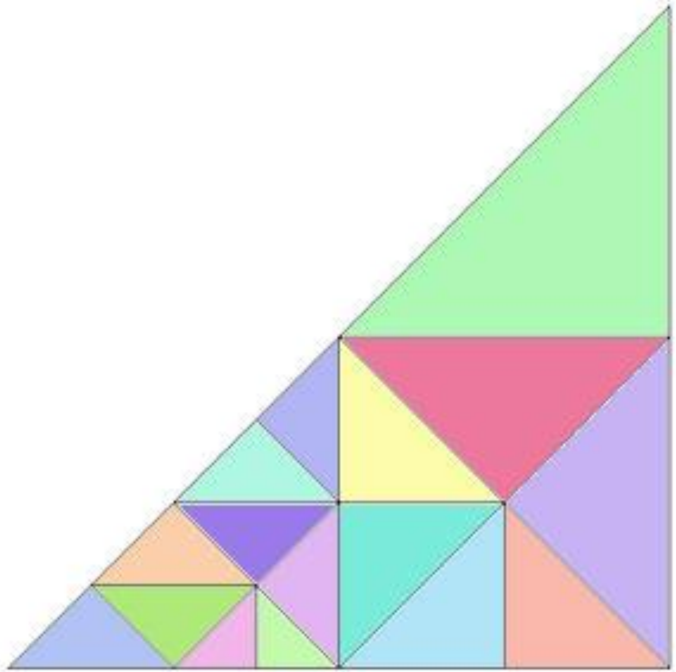


Proposition 2. Let $f(n)$ be the number of edge-to-edge n-tilings of a right isosceles triangle with right isosceles triangles. Then $f(n)$ is bounded by $\mathbf{2(3^{n-1})}$.

- Emily Caputo, Sophie Harteveldt, Alina Patwari, 14, 14, and 15

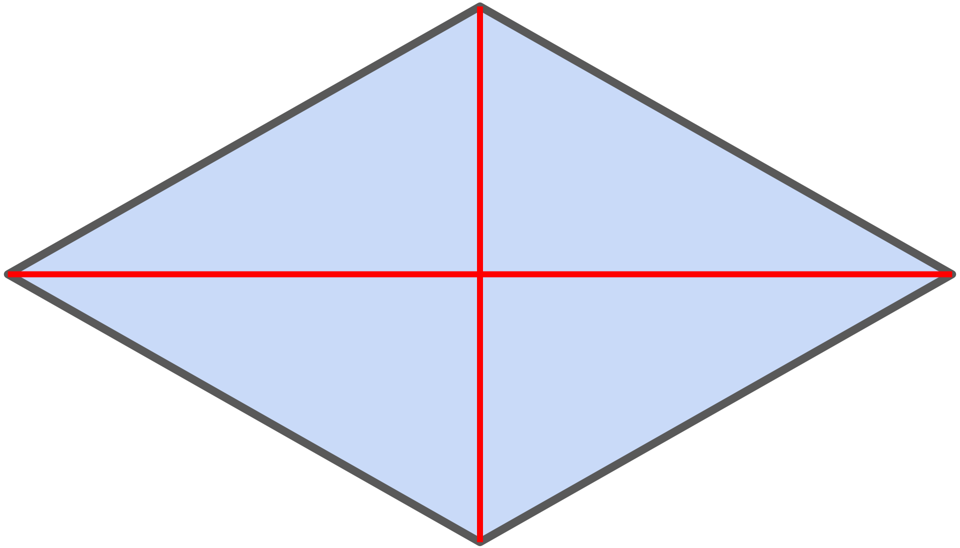
Proof (sketch). Proof (sketch): The three found an efficient algorithm for finding such tilings and their algorithm gives a way to encode each tiling as a string of *n* letters from a 3-letter alphabet.

For details, see E. Caputo, S. Harteveldt, A. Patwari, A Tiling Problem, *Girls' Angle Bulletin*, 17(1), 6-10 (2023).



The authors obtained further results which they presented at Boston College on April 9, 2024.

They are working on a paper and hope to submit it soon.

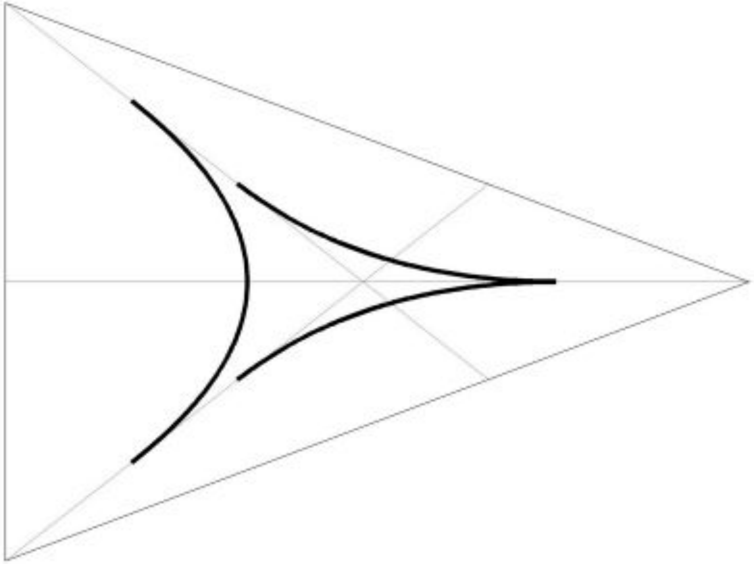


Proposition 3. Let Q be a quadrilateral with the property that each of its four angle bisectors split its perimeter in half. Then Q is either a rhombus or ________________________________?

Milena Harned

Proposition 3. Let Q be a quadrilateral with the property that each of its four angle bisectors split its perimeter in half. Then Q is either a rhombus or **a kite with three congruent acute angles**.

- Milena Harned (12 years old)



Milena Harned

Proof (Sketch). Study the envelope of the perimeter bisectors.

INTERNATIONAL JOURNAL OF GEOMETRY
Vol. 10 (2021), No. 4, 85 - 106

PERIMETER BISECTORS, CUSPS, AND KITES

MILENA HARNED

Abstract. We identify the specific convex quadrilaterals whose angle bisectors are also perimeter bisectors to be the rhombi and the kites with three congruent acute angles. The proof of this result uses the envelope of the lines that bisect the quadrilateral's perimeter. We also investigate and make some observations regarding these envelopes for more general convex polygons.

1. Introduction

Our main result is

Theorem 1.1. *Let P be a convex quadrilateral. The following are equivalent:*

- *Every angle bisector of P bisects its perimeter.*
- *P is either a kite with three congruent acute angles or a rhombus.*

To prove this result, we needed to consider the envelope of the perimeter

Milena Harned

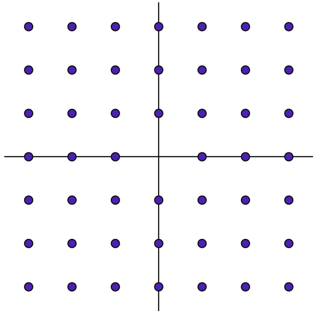
Perimeter bisectors, cusps, and kites, *International Journal of Geometry*, Vol. 10 (2021), No. 4, 85-106

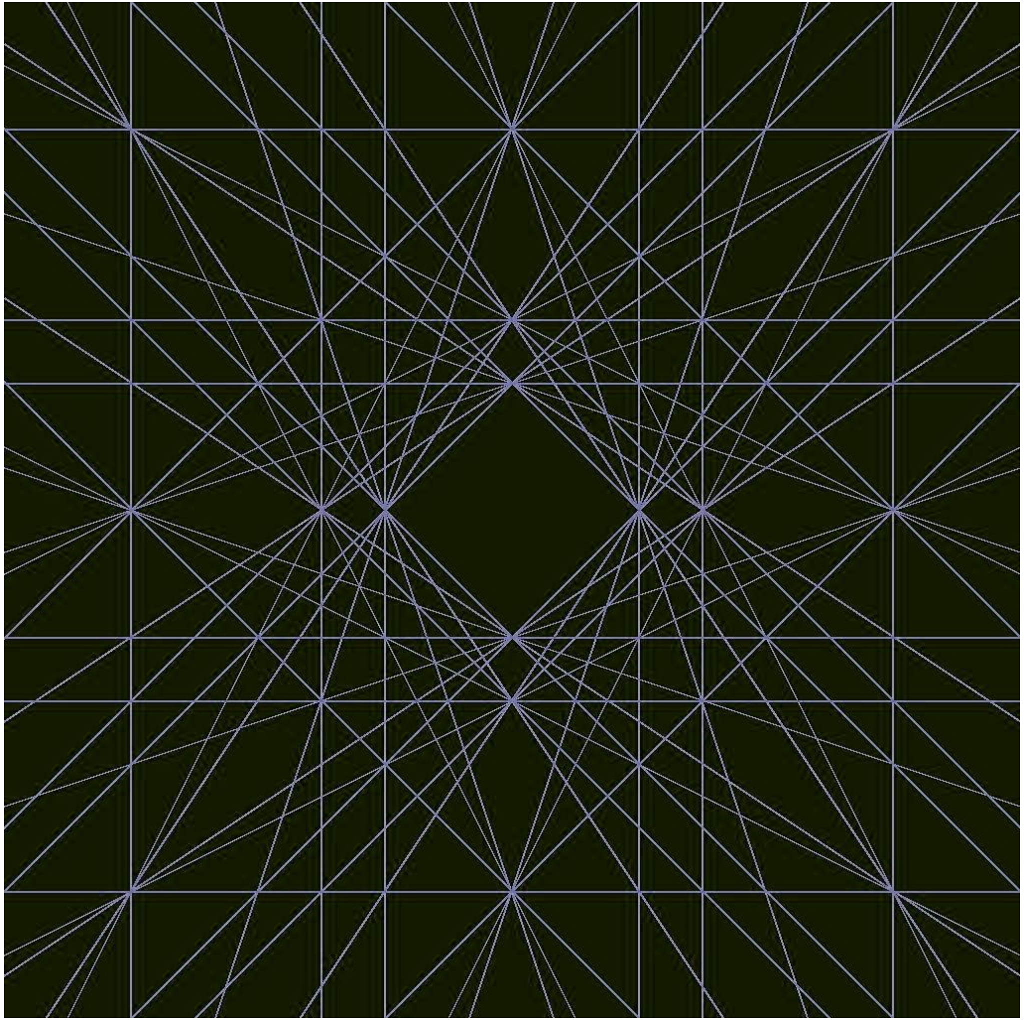
Proposition 4. Fix a positive integer N. Consider lines of the form $Ax + By = 1$, where A and B are integers such that

$$0 < |A + B| + |A - B| < N.$$

These lines partition the plane into convex regions that are ________________________?

$N = 7$



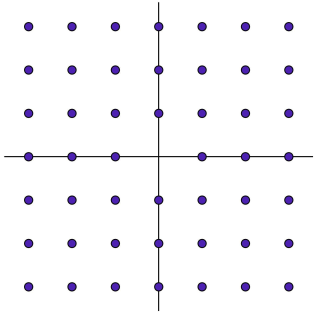


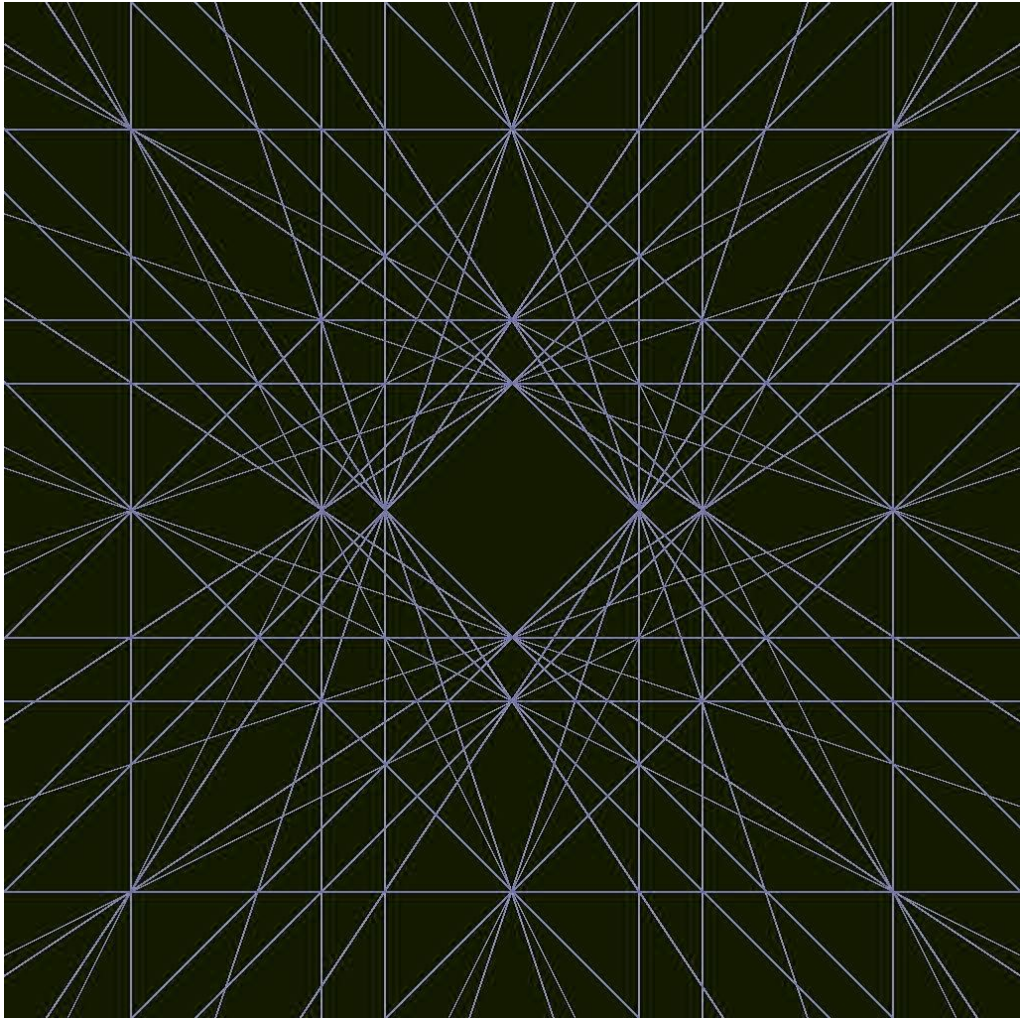
Proposition 4. Fix a positive integer N. Consider lines of the form $Ax + By = 1$, where A and B are integers such that

$$0 < |A + B| + |A - B| < N.$$

These lines partition the plane into convex regions that are **all triangles or quadrilaterals**.

- Milena Harned (13 years old) and Iris Liebman (13 years old)

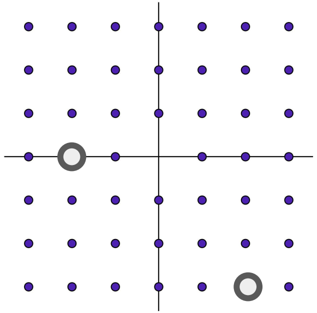


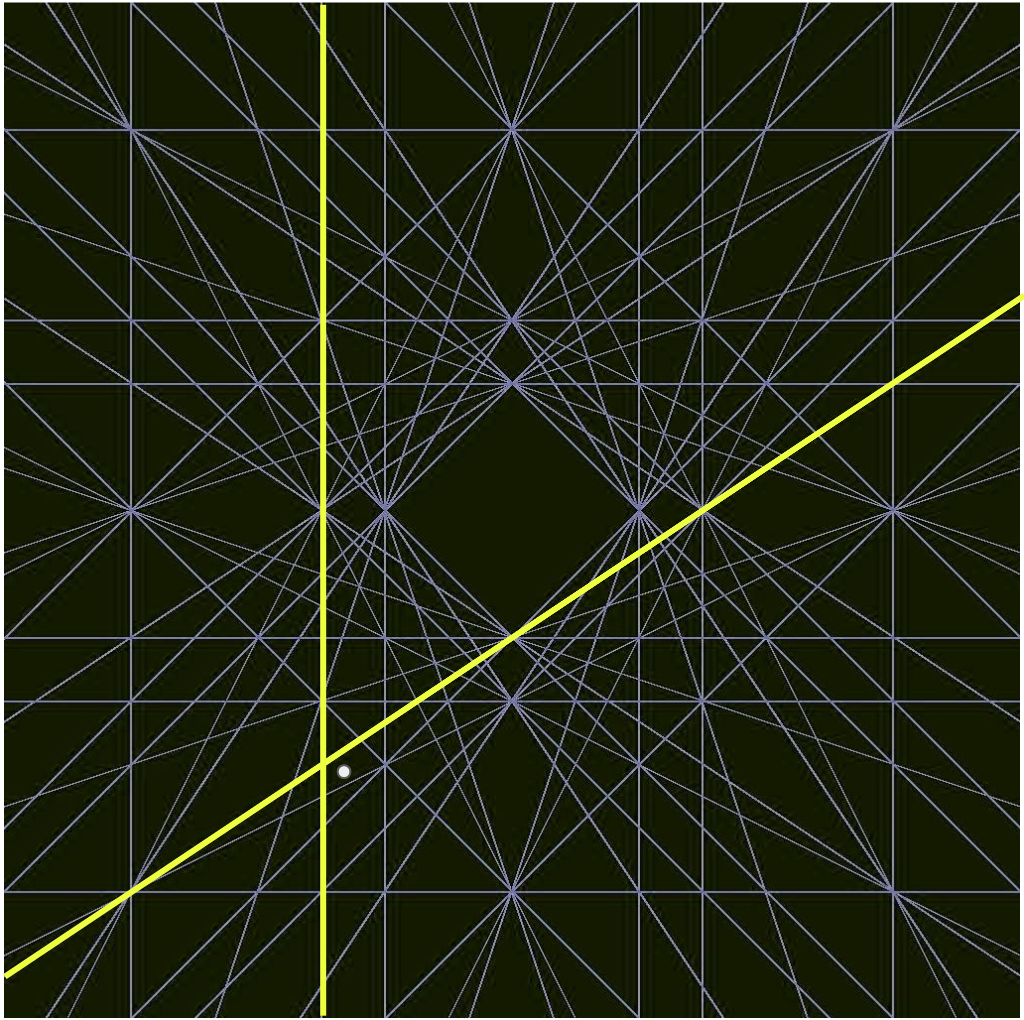


Proof (sketch). Let S be a finite set of points in the plane, excluding the origin.

For $P = (A, B)$ in S, let $L(P)$ be the line $Ax + By = 1$.

Given P and Q in S and a choice of one of the four angles formed by $L(P)$ and $L(Q)$, Harned and Liebman figured out how to find R in S such that $L(R)$ contains the next side around the region containing the angle.

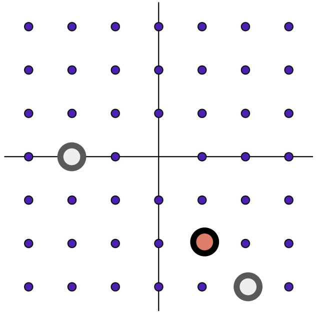


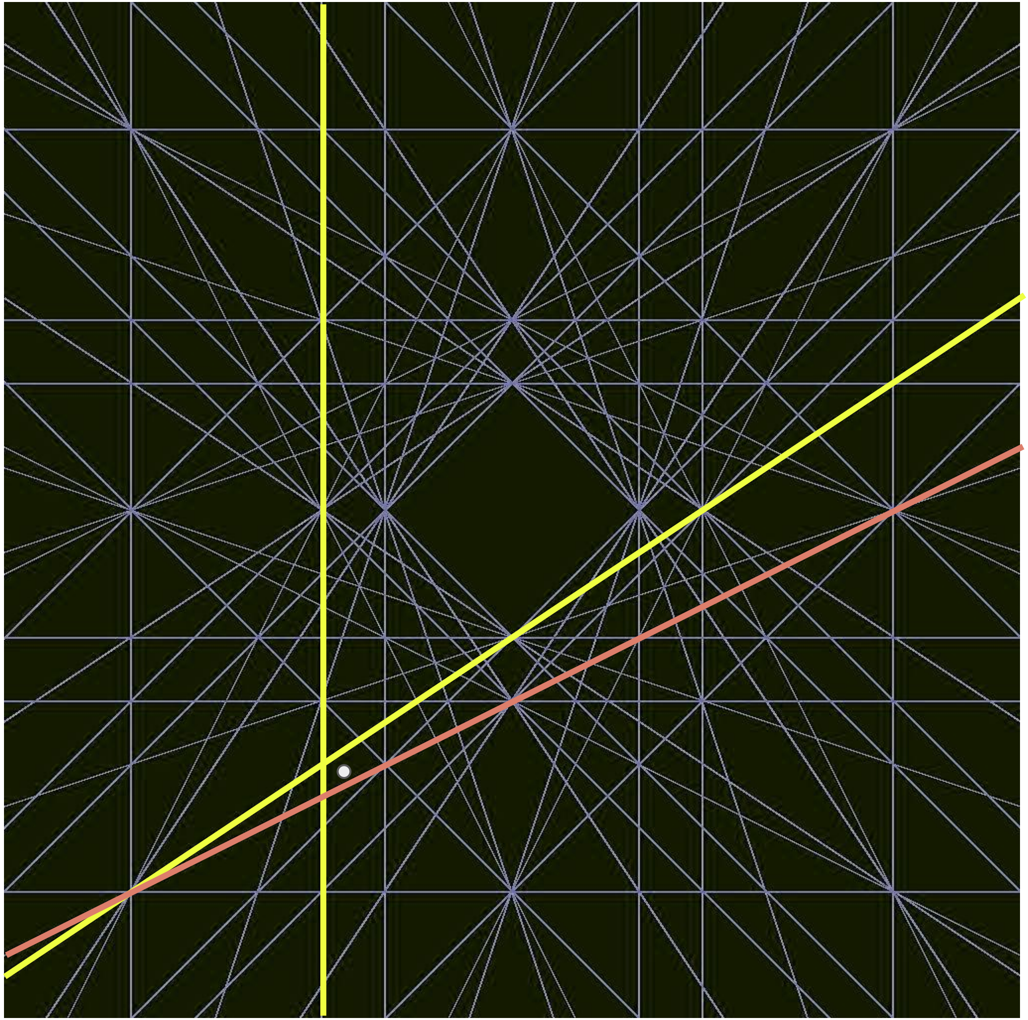


Proof (sketch). Let S be a finite set of points in the plane, excluding the origin.

For $P = (A, B)$ in S, let $L(P)$ be the line $Ax + By = 1$.

Given P and Q in S and a choice of one of the four angles formed by $L(P)$ and $L(Q)$, Harned and Liebman figured out how to find R in S such that $L(R)$ contains the next side around the region containing the angle.



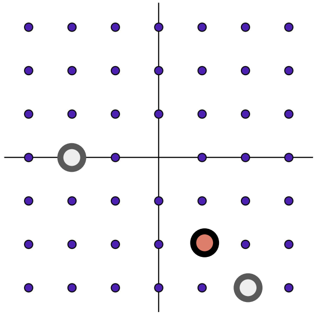


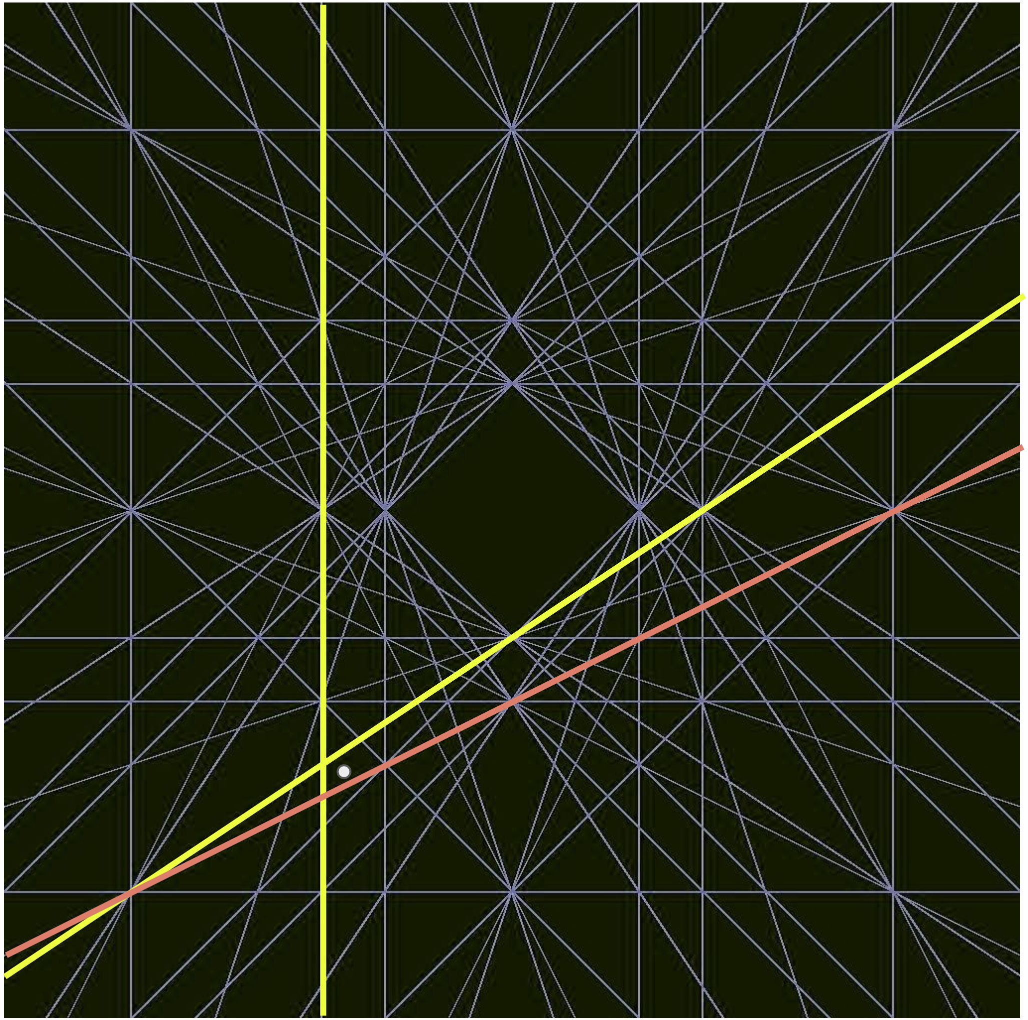
An Unexpected Class of 5+gon-free Line Patterns,

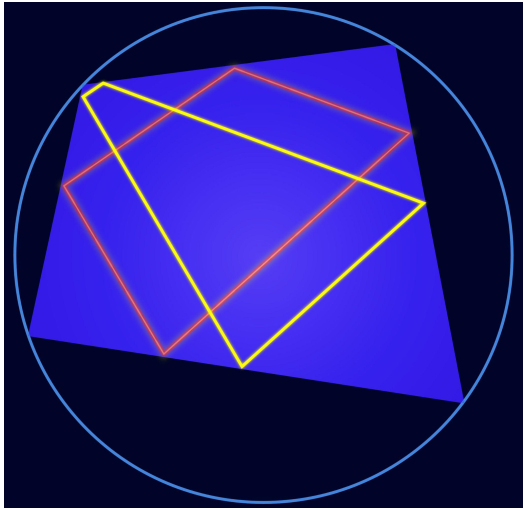
M. Harned and I. Liebman

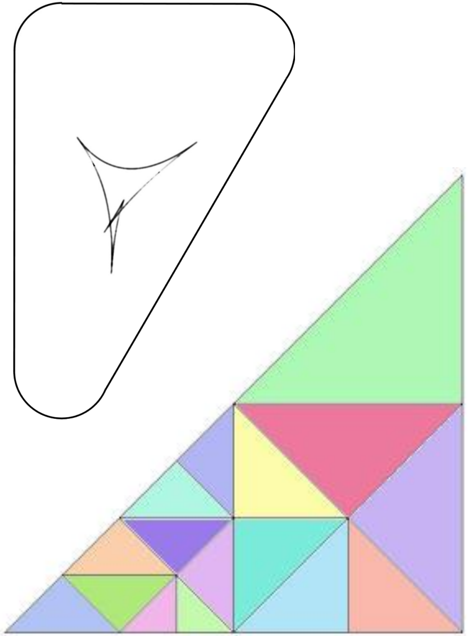
arXiv:2312.12447 [math.CO]

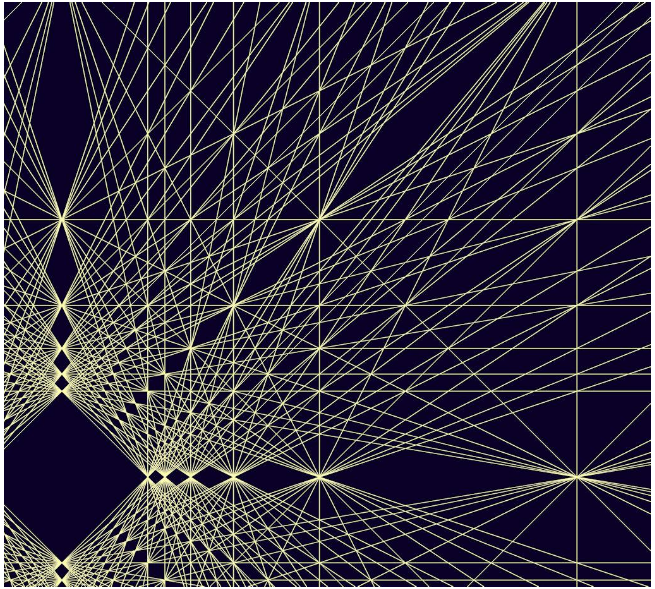
(Currently in the referee process.)











Theorem: Girls are fabulous mathematicians!

Proof: Propositions 1, 2, 3, and 4. QED!