$\mathfrak{g}$: cpx simple Lie alg of type ADE

- $\mathfrak{g}^{(1)}$: untwisted affine Lie alg. ass. with $\mathfrak{g}$
- choose diagram auto of $\mathfrak{g}$ and consider $\mathfrak{g}^{(r)}$: twisted affine Lie alg.
- ${}^{L}\mathfrak{g}^{(r)} = {}^{L}(\mathfrak{g}^{(r)})$: Langlands dual Lie alg. $\leftarrow$ untwisted affine Lie alg.

$\cap$

of type $A_{2n}^{(2)}$

$U_q(\mathfrak{g}^{(1)}),\ U_q(\mathfrak{g}^{(r)}),\ U_q({}^{L}\mathfrak{g}^{(r)})$: quantum affine algebras $\quad \circ\Rightarrow\circ\cdots\circ\Rightarrow\circ$

Vague Conjecture

Representation Theories of three algebras are "related".

evidences

- D. Hernandez '07 $\exists$ explicit ring isom. $K_0(Rep^{fin}(U_q(\mathfrak{g}^{(1)}))) \cong K_0(Rep^{fin}(U_q(\mathfrak{g}^{(r)})))$

sending KR-modules to KR modules

- E. Frenkel-Hernandez '09 proposed "Langlands duality" between $Rep^{\wedge}(U_q(\mathfrak{g}^{(r)}))$ & $Rep^{fin}(U_q({}^L\mathfrak{g}^{(r)}))$

deformation of q-characters $\exists$ interpolating objects

Kashiwara-Oh '17, Kashiwara-Kim-Oh '19

$\mathfrak{g} = A, D$ $\quad \mathrm{Rep}^{fin}(U_q(\mathfrak{g}^{(1)}))$ $\supset$

$r = 2$ $\quad \mathrm{Rep}^{fin}(U_q(\mathfrak{g}^{(r)}))$ $\supset$ Rep. KLR-alg. of type $\mathfrak{g}$

$\mathrm{Rep}^{fin}(U_q(L\mathfrak{g}^{(r)}))$ $\supset$ = convolution alg. of a quiver of type ADE

"large" monoidal subcategory

Lusztig's canonical base

(Schur-Weyl duality)

- Geometric approach to this problem, using quiver varieties.
- also category of possibly ∞-dimensional representations of truncated shifted quantum affine algebras $\leftrightarrow$ quantized Coulomb branch

Step 1 Modify the constructions of $\underbrace{U_q(\mathfrak{gl}_n)}_{\text{rep of}}$ to other cases.

Step 2. Analyse representation theories using technique of Ginzburg.

- compute fixed pt subvariety in quiver variety / variety of triples w.r.t. 1PS group in the torus acting on

Fixed pts are the same.

1. $\mathfrak{g}^{(r)}$ - case (proposed by Vasserot many years ago)

σ : diagram auto of $\mathfrak{g}$ $\sigma^r = 1$.

$\mathfrak{M}(v,w)$: quiver variety of type ADE

finite subgroup

$G = \prod_i GL(W_i)$

$$\bigoplus_v K^{G\times\mathbb{C}^\times \rtimes\langle\sigma\rangle}(\mathfrak{M}(v,w)) \otimes_{R(\langle\sigma\rangle)} \mathbb{C} \leftarrow \text{rep. of } U_q(\mathfrak{g}^{(1)})$$

$\mathfrak{g}^{(r)}$

evaluation at σ

Rem. This construction cannot be applied to equiv. homology group naively.

- cf. Lusztig canonical base for symmetrizable KM
- cf. Chriss affine Hecke alg with unequal param.

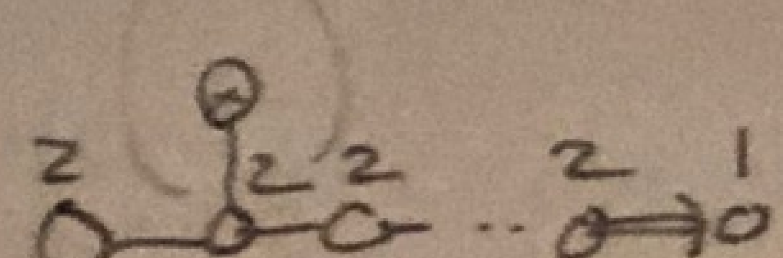
2. ${}^L\mathfrak{g}^{(r)}$ $\neq A^{(2)}_{2n}$

2 or 3-fold covering

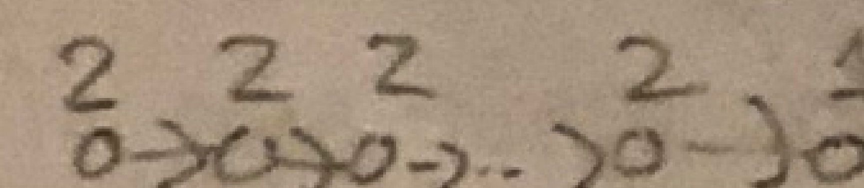
quantized Coulomb branch with A. Weekes

two cpx variables $z_s, z_\ell \in \mathbb{C}$

short long $z_s^{2 \text{ or } 3} = z_\ell$

$B_n^{(1)}$ 

← symmetrizer

Ignore $\Rightarrow$ and consider the quiver of type A 

$V = \bigoplus_i V_i$, $W = \bigoplus_i W_i$ $\quad N = \bigoplus_{i \to j} \mathrm{Hom}(V_i, V_j) \oplus \bigoplus_i \mathrm{Hom}(W_i, V_i)$

$\mathcal{R}$ = modified version of the variety of triples

= moduli space of

- bundles $\mathcal{E}_i$ over $\mathrm{Spec}\,\mathbb{C}[[z_s]]_{\cap\ell} = D_{s\cap\ell}$
- framing $\mathcal{E}|_{D_{s\cap\ell}\setminus\{0\}} \overset{\varphi_i}{\cong} \mathcal{O}_{D_{s\cap\ell}} \otimes_{\mathbb{C}} V_i$ $\quad \pi : D_s \to D_\ell$
- N-valued section $B_{i\to j} : \mathcal{E}_i \to \mathcal{E}_j$ $\quad$ pushforward by π

$a_i : \mathcal{O}_{D_{s\cap\ell}} \otimes_{\mathbb{C}} V_i \to \mathcal{E}_i$

s.t. their conjugates under φ_i extend across $0 \in D_{s\cap\ell}$

For $\mathfrak{g}^{(q)}$, the usual quantized Coulomb branch

$\mathcal{R}/_{\prod GL(V_i)[[z]]}$ = moduli sp. of bdle + sections over  $= D \cup_{D^\times} D$

= space of maps from  $\to \left[N/_{\prod GL(V_i)} \right]$

For ${}^L\mathfrak{g}^{(m)}$: we replace <u>domain</u>, so it is not simply the space of maps to a modified space

$\mathcal{R} \curvearrowleft G = \prod GL(V_i)[[z]]$

$H_*, K^{G \rtimes \mathbb{C}^\times_{\mathrm{rot}}}(\mathcal{R})$ $\qquad \mathbb{C}^\times_{\mathrm{rot}} \ni t \qquad z_i \mapsto t z_i$

$\qquad z_i \mapsto t^{2d_i} z_i$

Th. [N-Weekes]

(1) $H^G_*(\mathcal{R})$ is commutative. $\mathcal{M}_C = \operatorname{Spec} H^G_*(\mathcal{R})$ Coulomb branch

is generalized slice of affine Grassmannian
sym. subv. of $\mathcal{M}_C$ of type $BCFG$

realize rep. of $\mathfrak{g}_{BCFG}$

geom. Satake

(2) $H_*^{G\times\mathbb{C}^\times}(\mathcal{R}) \cong$ truncated shifted Yangian of type BCFG

$K^{G\times\mathbb{C}^\times}(\mathcal{R}) \underset{?}{\cong}$ " quantum affine algebra for ${}^L\mathfrak{g}^{(r)}$.

$\mathbb{C}^\times \to \prod GL(V_i)$

$\mathcal{T} = \mathcal{R}$ but without ext. condition

$\downarrow$

N_K = N-valued sections of trivial bdles over D

Apply analysis of the convolution algebras

$\left(\mathcal{T} \times_{N_K} \mathcal{T}\right)^{1\text{-ps}}$ = variety appeared in Lusztig's work on canonical base (ADE)