Some Comments on the Structure of the Unitary Dual

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Problem of Unitary Dual

Let $G$ be a complex connected reductive algebraic group. Write

$$\Pi_{u,sph}(G) = \{\text{irred unitary spherical } G\text{-representations}\}.$$

Problem of the Unitary Dual (complex spherical case)

Parameterize the set $\Pi_{u,sph}(G)$.

Some history:

- $\text{GL}(2)$ (Gelfand-Naimark, 1947)
- $\text{SL}(3), \text{Sp}(4), G_2$ (Duflo, 1979)
- $\text{GL}(n)$ (Vogan, 1986)
- $\text{Sp}(2n), \text{SO}(n)$ (Barbasch, 1989)

Goal: give a conjectural description of $\Pi_{u,sph}(G)$ for all $G$. 
Harish-Chandra bimodules

- A Harish-Chandra bimodule is a $U(g)$-bimodule $V$ such that the adjoint action of $g$

  $$g \times V \to V, \quad (\xi, v) \mapsto \xi v - v\xi$$

  integrates to a rational (i.e. locally finite) $G$-action.

- A HC bimodule is spherical if it contains a nonzero fixed vector for the adjoint $G$-action.

- Write

  $$HC(G) = \{\text{irred HC bimodules}\}$$
  $$HC_{sph}(G) = \{\text{irred spherical HC bimodules}\}$$
Fix a *compact* real form $\sigma : \mathfrak{g} \to \mathfrak{g}$. Induces a conjugate-linear algebra involution $\sigma : U(\mathfrak{g}) \to U(\mathfrak{g})$.

A Hermitian form $\langle \cdot , \cdot \rangle$ on a HC bimod $V$ is *invariant* if

$$\langle xvy, w \rangle = \langle v, \sigma(y)w\sigma(x) \rangle, \quad x, y \in U(\mathfrak{g}), \; v, w \in V.$$

$V$ is *Hermitian* if it admits a non-degenerate invariant Hermitian form.

$V$ is *unitary* if it admits a positive-definite invariant Hermitian form.

Write

$$HC_u(G) = \{\text{irred unitary HC bimodules}\}$$

$$HC_{u, sph}(G) = \{\text{irred spherical unitary HC bimodules}\}$$
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Harish-Chandra Bimodules

Theorem (Harish-Chandra, Duflo,...)

\[
\begin{align*}
\Pi_{u,sph}(G) & \leftrightarrow \Pi_{sph}(G) \leftrightarrow \Pi(G) \\
\downarrow & \quad \downarrow & \quad \downarrow & \quad \downarrow \\
HC_{u,sph}(G) & \leftrightarrow HC_{sph}(G) \leftrightarrow HC(G) \\
\downarrow & \quad \downarrow \\
\frak{h}^* / \mathcal{W} & 
\end{align*}
\]
Thus, we can regard $\Pi_{u,sph}(G)$ as a $W$-invariant subset of $\mathfrak{h}^*$. 

Problem of Unitary Dual (algebraic formulation, complex spherical case)

Compute the $W$-invariant subset $\Pi_{u,sph}(G) \subset \mathfrak{h}^*$.

Remark

It is useful and customary to restrict to the case of ‘real infinitesimal character’, i.e. $X^*(H) \otimes_{\mathbb{Z}} \mathbb{R} \subset \mathfrak{h}^*$. One can easily reduce to this case via unitary induction.
Some general features of $\Pi_{u, sph}(G)$:

- It is a \textit{closed} subset of $\mathfrak{h}^*$ (in the Euclidean topology).
- It is contained in the closed ball $B(0, |\rho|)$ (probably a tighter bound is possible).
- It is a union of facets defined by certain hyperplanes in $\mathfrak{h}^*$ (roughly: affine co-root hyperplanes).

Ok, but what does it look like?
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$SL(2, \mathbb{C})$
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$Sp(4, \mathbb{C})$
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$G_2(\mathbb{C})$
How should we understand these pictures?

(1) Each picture contains a finite set of distinguished points:
For each $G$, there is a finite set of reps (e.g. trivial, oscillator rep) called *unipotent representations*, which are unitary for magical reasons.

(2) Each picture contains copies of the pictures for its Levis:
If $L \subset G$ is a Levi subgroup and $X_L \in \Pi_{u,sph}(L)$, then $\text{Ind}_P^G X_L$ is unitary (and hence also its spherical summand).

(3) Each picture is closed under certain ‘deformations’: If $X \in \Pi_{u,sph}(G)$ belongs to a ‘complementary series’ $C$, then $C \subset \Pi_{u,sph}(G)$.

Vogan’s Philosophy on the Unitary Dual (‘Orange Book’, 1987)

Every representation in $\Pi_{u,sph}(G)$ can be obtained by applying operations (2) and (3) to a unipotent representation (1) of a Levi subgroup $L \subset G$. 
Vogan’s Philosophy

In order to turn Vogan’s philosophy into a precise mathematical conjecture, we need:

- a precise (and suitably general) definition of ‘unipotent’, and
- a precise (and suitably general) definition of ‘complementary series’.

Claim: both goals are most naturally accomplished using the language of filtered quantizations of nilpotent covers.
Nilpotent covers

- A nilpotent cover for $G$ is a finite, connected, $G$-equivariant cover of a nilpotent co-adjoint $G$-orbit.

Write

$$\text{Cov}(G) = \{\text{nilpotent covers for } G\}/\sim$$

- If $\mathcal{O}$ is a nilpotent orbit and $e \in \mathcal{O}$, then covers of $\mathcal{O}$ are parameterized by conjugacy classes of subgroups of $A(\mathcal{O}) = Z_G(e)/Z_G(e)^\circ$.

Example: $\text{SL}(2, \mathbb{C})$

Two nilpotent orbits: $\{0\}$ and $\mathcal{O} = G \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

- $A(\{0\}) = 1$. No nontrivial covers.
- $A(\mathcal{O}) = \mathbb{Z}_2$. One nontrivial (two-fold) cover.
Birational induction of nilpotent covers

- For each Levi subgroup $L \subset G$, there is a map 
  $$\text{Bind}_{L}^{G} : \text{Cov}(L) \rightarrow \text{Cov}(G)$$
  called *birational induction*.

- A cover is said to be *birationally rigid* if it cannot be obtained via birational induction from a proper Levi subgroup.

- A *birational induction datum* is a pair $(L, \tilde{O}_{L})$ consisting of a Levi subgroup $L \subset G$ and a birationally rigid nilpotent cover $\tilde{O}_{L}$. Write
  $$\Psi(G) = \{\text{birational induction data }(L, \tilde{O}_{L})\}$$

**Proposition (Losev, Matvieieievskyi)**

$$\text{Bind} : \Psi(G)/G \xrightarrow{\sim} \text{Cov}(G).$$
Quantizations of nilpotent covers

The ring of regular functions $\mathbb{C}[\tilde{O}]$ is a graded Poisson algebra. Can define *filtered quantizations* of $\mathbb{C}[\tilde{O}]$. Write

$$Q(\tilde{O}) := \{\text{filtered quantizations of } \mathbb{C}[\tilde{O}]\}/\sim$$

Choose $(L, \tilde{O}_L) \in \Psi(G)$ corresponding to $\tilde{O}$, and define

$$\mathfrak{h}(\tilde{O}) := \mathfrak{z}(L \cap [g,g])^*$$

**Theorem (Losev, Losev-MB-Matvieievskyi)**

There is a (finite) subgroup $W(\tilde{O}) \subset N_G(L)/L$ and a canonical bijection

$$\mathfrak{h}(\tilde{O})/W(\tilde{O}) \xrightarrow{\sim} Q(\tilde{O}), \quad \lambda \mapsto A_\lambda(\tilde{O})$$
Example

- Let $P = LU \subset G$ be a parabolic subgroup.
- There is a unique open $G$-orbit

$$\tilde{O} \subset T^*(G/P).$$

- Image of the moment map $T^*(G/P) \to \mathcal{N}$ is the closure of a nilpotent orbit (Richardson orbit for $P$).
- $\tilde{O}$ is a nilpotent cover, birationally induced from $(L, \{0\})$.
- $\mathfrak{h}(\tilde{O}) = z(I \cap [g, g])^*$.
- Given $\lambda \in \mathfrak{h}(\tilde{O})$, get TDO $\mathcal{D}^{\lambda+\rho(u)}_{G/P}$ on $G/P$. Then

$$\mathcal{A}_{\lambda}(\tilde{O}) = \Gamma(G/P, \mathcal{D}^{\lambda+\rho(u)}_{G/P})$$
Quantizations of nilpotent covers

Proposition (Losev-MB-Matwieieievskyi)

For each $A_\lambda(\tilde{O}) \in Q(\tilde{O})$, there is a unique quantum co-moment map

$$\Phi : U(g) \to A_\lambda(\tilde{O})$$

such that $\Phi|_{\tilde{\delta}(g)} = 0$. The map $\Phi$ turns $A_\lambda(\tilde{O})$ into a finite-length, spherical Harish-Chandra bimodule for $U(g)$. Write

$$I_\lambda(\tilde{O}) := \ker(\Phi).$$

This is a completely prime, primitive ideal.

Definition (Losev-MB-Matwieieievskyi)

The unipotent ideal attached to $\tilde{O}$ is $I_0(\tilde{O})$. 
Take $\tilde{\mathfrak{O}} \in \text{Cov}(G)$ corresponding to $(L, \tilde{\mathfrak{O}}_L) \in \Psi(G)$.

For each $A_\lambda(\tilde{\mathfrak{O}}) \in Q(\tilde{\mathfrak{O}})$, write

$$\gamma_\lambda(\tilde{\mathfrak{O}}) = \text{infl char of } l_\lambda(\tilde{\mathfrak{O}}) \in \mathfrak{h}^*/W.$$  

**Lemma (Losev-MB-Matvieievsyki)**

$$\gamma_\lambda(\tilde{\mathfrak{O}}) = \gamma_0(\tilde{\mathfrak{O}}_L) + \lambda.$$  

This reduces the calculation of $\gamma_\lambda(\tilde{\mathfrak{O}})$ to the calculation of $\gamma_0(\tilde{\mathfrak{O}})$ for birationally rigid covers. The latter calculation was carried out in Losev-MB-Matvieievsyki (classical groups) and MB-Matvieievsyki (spin and exceptional groups).
Simple quantizations of nilpotent covers

When is $A_\lambda(\mathfrak{O})$ a simple algebra?

**Theorem (Losev-MB-Matwieievskyi)**

- The algebra $A_\lambda(\mathfrak{O})$ is simple if and only if the ideal $I_\lambda(\mathfrak{O})$ is maximal.
- The ideal $I_\lambda(\mathfrak{O})$ is maximal if and only if $\gamma_\lambda(\mathfrak{O})$ satisfies a straightforward combinatorial condition.
- This combinatorial condition is satisfied in an open subset of $\mathfrak{h}(\mathfrak{O})$ (including 0).

Examples later...
Real structures on quantizations of nilpotent covers

- Let $\sigma$ be a compact form of $\mathfrak{g}$.
- If $\mathcal{O}$ is a nilpotent orbit, then $\sigma$ preserves $\mathcal{O}$, induces a real form $\sigma$ on $\mathbb{C}[\mathcal{O}]$.
- A cover $\tilde{\mathcal{O}}$ is said to be *relevant* if it is birationally induced from a nilpotent orbit.
- If $\tilde{\mathcal{O}}$ is relevant, then $\sigma$ induces a real form $\sigma$ on $\mathbb{C}[\tilde{\mathcal{O}}]$.
- A quantization $\mathcal{A}_\lambda(\tilde{\mathcal{O}})$ of a relevant cover is *real* if $\sigma$ lifts to a (necessarily unique) real form on $\mathcal{A}_\lambda(\tilde{\mathcal{O}})$.
- If $\tilde{\mathcal{O}}$ is relevant, then $\mathcal{A}_\lambda(\tilde{\mathcal{O}})$ is real if and only if
  \[ -\lambda \in W(\tilde{\mathcal{O}}) \lambda \]
Let $A_\lambda(\tilde{O})$ be a real quantization of a relevant cover and let $V$ be a Harish-Chandra $A_\lambda(\tilde{O})$-bimodule.

- A Hermitian form $\langle \ , \ \rangle$ on $V$ is invariant if
  \[ \langle xvy, w \rangle = \langle v, \sigma(y)w\sigma(x) \rangle, \quad x, y \in A_\lambda(\tilde{O}), \ v, w \in V. \]

- $V$ is Hermitian if it admits a non-degenerate invariant Hermitian form.

- $V$ is unitary if it admits a positive-definite invariant Hermitian form.

- If $V$ is Hermitian/unitary as a $A_\lambda(\tilde{O})$ bimodule, it is Hermitian/unitary as a $U(g)$-bimodule.

- If $\Phi : U(g) \to A_\lambda(\tilde{O})$ is surjective, then the converse is also true.
Hermitian quantizations of nilpotent covers

Let $A_\lambda(\tilde{O})$ be a real quantization of a relevant cover.

- $A_\lambda(\tilde{O})$ contains a *unique* copy of the trivial representation. Consider the projection

  $$\eta : A_\lambda(\tilde{O}) \to \mathbb{C}$$

- Define a Hermitian form on $A_\lambda(\tilde{O})$ by

  $$\langle x, y \rangle := \eta(x\sigma(y))$$

**Proposition**

- $\langle \ , \ \rangle$ is invariant.
- $\langle \ , \ \rangle$ is the *unique* invariant Hermitian form on $A_\lambda(\tilde{O})$.
- $\langle \ , \ \rangle$ is non-degenerate if and only if $A_\lambda(\tilde{O})$ is simple.
Induction of quantizations of nilpotent covers

Suppose \( \widetilde{\mathcal{O}} \) corresponds to \((L, \widetilde{\mathcal{O}}_L) \in \Psi(G)\). Choose a Levi subgroup \( M \subset G \) containing \( L \). Define

\[
\widetilde{\mathcal{O}}_M := \text{Bind}^L_M \widetilde{\mathcal{O}}_L \in \text{Cov}(M).
\]

Can define \textit{parabolic induction} for filtered quantizations

\[
\text{Ind}^G_M : Q(\widetilde{\mathcal{O}}_M) \to Q(\widetilde{\mathcal{O}}).
\]

Corresponds to the natural inclusion, on the level of parameters

\[
\mathfrak{h}(\widetilde{\mathcal{O}}_M) = \mathfrak{z}(l \cap [m, m])^* \hookrightarrow \mathfrak{z}(l \cap [g, g])^* = \mathfrak{h}(\widetilde{\mathcal{O}}).
\]

- If \( A_\lambda(\widetilde{\mathcal{O}}_M) \) is real, then \( \text{Ind}^G_M A_\lambda(\widetilde{\mathcal{O}}_M) \) is real.
- If \( A_\lambda(\widetilde{\mathcal{O}}_M) \) is unitary, then \( \text{Ind}^G_M A_\lambda(\widetilde{\mathcal{O}}_M) \) may not be Hermitian (i.e. simple), but if it is Hermitian, it is automatically unitary.
Complementary series for quantizations of nilpotent covers

Let \( \tilde{\mathcal{O}} \) be a relevant cover. Write:

\[
Q(\tilde{\mathcal{O}}) \
\cup \\
Q_R(\tilde{\mathcal{O}}) = \{ \text{real quantizations of } \mathbb{C}[\tilde{\mathcal{O}}] \} \\
\cup \\
Q_h(\tilde{\mathcal{O}}) = \{ \text{Hermitian quantizations of } \mathbb{C}[\tilde{\mathcal{O}}] \} \\
\cup \\
Q_u(\tilde{\mathcal{O}}) = \{ \text{unitary quantizations of } \mathbb{C}[\tilde{\mathcal{O}}] \}
\]

Write \( h_R(\tilde{\mathcal{O}}), h_h(\tilde{\mathcal{O}}), h_u(\tilde{\mathcal{O}}) \) for the corresponding parameter spaces. Recall

\[
Q_h(\tilde{\mathcal{O}}) = \{ A \in Q_R(\tilde{\mathcal{O}}) \mid A \text{ simple} \}
\]
The set $h_h(\tilde{\mathbb{O}})$ decomposes into connected components. If $S \subset h_h(\tilde{\mathbb{O}})$, define

$$C(S) = \text{union of all connected components which meet } S \text{ nontrivially.}$$

This induces an operation on $Q_h(\tilde{\mathbb{O}})$.

**Proposition**

If $S \subset Q_u(\tilde{\mathbb{O}})$, then $C(S) \subset Q_u(\tilde{\mathbb{O}})$.

Note: some quantizations in the family $C(S)$ may be reducible as $U(g)$-bimodules. So $C(S)$ may extend the usual complementary series.
Conjectures

Conjecture

Suppose $\tilde{\Omega}$ is relevant. Then

$$Q_u(\tilde{\Omega}) = C(Q_h(\tilde{\Omega}) \cap \bigcup_{M \supseteq L} \text{Ind}_M^G Q_u(\tilde{\Omega}_M)).$$

Conjecture

$$\Pi_{u,sph}(G) = \bigcup \{ U(\mathfrak{g})/I_\lambda(\tilde{\Omega}) \mid A_\lambda(\tilde{\Omega}) \in Q_u(\tilde{\Omega}) \}.$$
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$Sp(4, \mathbb{C})$: principal orbit

Figure: $h(\tilde{O})$
$Sp(4, \mathbb{C})$: principal orbit

**Figure:** $\mathfrak{h}_R(\mathcal{O})$
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$\text{Sp}(4, \mathbb{C}):$ principal orbit

**Figure:** $\mathfrak{h}_h(\widetilde{\mathbb{O}})$
$\text{Sp}(4, \mathbb{C})$: principal orbit

Figure: $\mathfrak{h}_h(\tilde{\mathcal{O}})$
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\[ \text{Sp}(4, \mathbb{C}): \text{principal orbit} \]

Figure: \( \mathfrak{h}_u(\tilde{Q}) \)
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\[ \text{Sp}(4, \mathbb{C}): \text{subregular orbit} \]

**Figure:** \( \mathfrak{h}(\widetilde{\Omega}) \)
$\text{Sp}(4, \mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_R(\tilde{\mathcal{O}})$
$\text{Sp}(4, \mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_h(\tilde{O})$
$\text{Sp}(4, \mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_h(\tilde{Q})$
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$\text{Sp}(4, \mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_u(\tilde{O})$
$Sp(4, \mathbb{C})$: double cover of subregular orbit

Figure: $\mathfrak{h}(\tilde{\mathfrak{o}})$
$Sp(4, \mathbb{C})$: double cover of subregular orbit

Figure: $\mathfrak{h}_R(\tilde{O})$
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$\text{Sp}(4, \mathbb{C})$: double cover of subregular orbit

$\mathfrak{h}_h(\widetilde{O})$

Figure: $\mathfrak{h}_h(\widetilde{O})$
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$\text{Sp}(4, \mathbb{C})$: double cover of subregular orbit

Figure: $\mathfrak{h}_u(\mathcal{O})$
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$\text{Sp}(4, \mathbb{C})$: minimal orbit

Figure: $\mathfrak{h}(\hat{0})$
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$\textit{Sp}(4, \mathbb{C})$: minimal orbit

Figure: $\mathfrak{h}_R(\tilde{\mathcal{O}})$
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$\text{Sp}(4, \mathbb{C})$: minimal orbit

Figure: $\mathfrak{h}_h(\widetilde{O})$
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$\text{Sp}(4, \mathbb{C}):$ minimal orbit

Figure: $\mathfrak{h}_u(\tilde{O})$
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\[ \text{Sp}(4, \mathbb{C}): \text{zero orbit} \]

\[ \text{Figure: } \tilde{h}(\tilde{0}) \]
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$Sp(4, \mathbb{C})$: zero orbit

Figure: $\mathfrak{h}_R(\tilde{\mathcal{O}})$
$\text{Sp}(4, \mathbb{C}):$ zero orbit

Figure: $h_h(\tilde{\Omega})$
$\text{Sp}(4, \mathbb{C})$: zero orbit

Figure: $\mathfrak{h}_u(\mathcal{O})$
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$\text{Sp}(4, \mathbb{C})$: putting it all together
Sp(4, \mathbb{C}): putting it all together
$G_2(\mathbb{C})$: principal orbit
$G_2(\mathbb{C})$: principal orbit

Figure: $\mathfrak{h}_R(\widetilde{\mathfrak{g}})$
$G_2(\mathbb{C})$: principal orbit

Figure: $\mathfrak{h}_h(\widehat{\Omega})$
$G_2(\mathbb{C})$: principal orbit

Figure: $\mathfrak{h}_h(\tilde{O})$
$G_2(\mathbb{C})$: principal orbit

Figure: $\mathfrak{h}_u(\mathfrak{O})$
$G_2(\mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}(\tilde{O})$
$G_2(\mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_R(\tilde{O})$
$G_2(\mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_h(\tilde{\mathcal{O}})$
$G_2(\mathbb{C})$: subregular orbit

Figure: $\mathfrak{h}_h(\mathbb{O})$
$G_2(\mathbb{C})$: subregular orbit

**Figure**: $\mathfrak{h}_u(\tilde{O})$
$G_2(\mathbb{C})$: three-fold cover of subregular orbit

Figure: $\mathfrak{h}(\sim O)$
$G_2(\mathbb{C})$: three-fold cover of subregular orbit

**Figure:** $\mathfrak{h}_R(\tilde{O})$
$G_2(\mathbb{C})$: three-fold cover of subregular orbit

Figure: $h_h(\tilde{O})$
$G_2(\mathbb{C})$: three-fold cover of subregular orbit

**Figure:** $\mathfrak{h}_h(\hat{O})$
$G_2(\mathbb{C})$: three-fold cover of subregular orbit

Figure: $\mathfrak{h}_u(\widehat{\Omega})$
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$G_2(\mathbb{C})$: 8-dim rigid orbit

Figure: $h(\tilde{O})$
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$G_2(\mathbb{C})$: 8-dim rigid orbit

Figure: $\mathfrak{h}_R(\widehat{O})$
$G_2(\mathbb{C})$: 8-dim rigid orbit

Figure: $h_h(\mathfrak{O})$
$G_2(\mathbb{C})$: 8-dim rigid orbit

Figure: $\mathfrak{h}_u(\widetilde{\mathfrak{O}})$
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$G_2(\mathbb{C})$: minimal orbit

Figure: $h(\tilde{O})$
$G_2(\mathbb{C})$: minimal orbit

**Figure:** $\mathfrak{h}_\mathbb{R}(\hat{0})$
$G_2(\mathbb{C})$: minimal orbit

Figure: $\mathfrak{h}_h(\widehat{\mathcal{O}})$
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\( G_2(\mathbb{C}) \): minimal orbit

Figure: \( \mathfrak{h}_u(\tilde{\mathcal{O}}) \)
$G_2(\mathbb{C})$: zero orbit

Figure: $\mathfrak{h}(\tilde{O})$
$G_2(\mathbb{C})$: zero orbit

Figure: $\mathfrak{h}_R(\tilde{\mathfrak{o}})$
$G_2(\mathbb{C})$: zero orbit

Figure: $\mathfrak{h}_h(\tilde{\mathfrak{o}})$
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$G_2(\mathbb{C})$: zero orbit

Figure: $\mathfrak{h}_u(\widehat{\mathfrak{o}})$
$G_2(\mathbb{C})$: putting it all together
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$G_2(\mathbb{C})$: putting it all together