REPRESENTATION THEORY, GEOMETRY, AND QUANTIZATION: THE MATHEMATICAL LEGACY OF BERTRAM KOSTANT
ABSTRACTS

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Joseph Bernstein
Tel Aviv University
On Langlands parameters in the local Langlands correspondence

Let $\mathcal{G}$ be a split reductive group over a p-adic field F, and G the group of its F-points.

The main insight of local Langlands program is that to every irreducible smooth representation (ρ, G, V) should correspond a morphism $\nu_\rho : W_F \to {}^\vee G$ of the Weil group W_F of the field F to the Langlands' dual group ${}^\vee G$.

Ideally this should extend to a bijection between the set $\mathrm{Irr}(G)$ of irreducible representations of G and some set $\mathrm{Lan}(G)$ of Langlands parameters for G that is described in terms of the dual group ${}^\vee G$. Even better would be to describe the category $\mathrm{Rep}(G)$ of smooth G-modules in terms of the dual group.

By now it is clear that to achieve these goals one should modify both the left hand side $\mathrm{Rep}(G)$ and the right hand side.

In my talk I will try to explain that in fact, starting from the first principles, it is clear that it is better to work not with the Langlands' dual group ${}^\vee G$ but with some slight modification of it. In fact this is necessary if we would like to have a **canonical** Langlands correspondence.

For example, in case of the group $G = PGL(2, F)$ one should assign to an irreducible representation ρ of the group G a morphism

$$\nu = \nu_\rho : W_F \to GL(2, \mathbf{C})$$

that satisfies the condition $\det(\nu) = \omega$, where $\omega : W_F \to \mathbf{C}^*$ is the cyclotomic character.

I will try to describe this modification and discuss different possible formulations of Local Langlands correspondence.

I will also discuss how this modification affects the notion of L-functions corresponding to an automorphic representation π. In particular, we will see that in this approach it is quite clear where to look for the special values of these L-functions.

Giovanni Felder
ETH
Gauge theory partition functions, representation schemes and random matrices

Nekrasov's partition functions of $N = 2$ supersymmetric gauge theories are generating series of integrals of equivariant cohomology or K-theory classes on instanton moduli spaces for all instanton numbers. They also arise in conformal field theory and derived representation schemes, and have a representation as random matrix integrals. I will review these relations and present recent results obtained with Martin Müller-Lennert on the radius of convergence of Nekrasov's partition functions and Whittaker vectors for the q-Virasoro algebra.

Victor Ginzburg
University of Chicago
Differential operators on G/U and the Gelfand-Graev action

Let G be a complex semisimple group and U its maximal unipotent subgroup. We study the algebra $D(G/U)$ of algebraic differential operators on G/U and also its quasi-classical counterpart: the algebra of regular functions on the cotangent bundle. A long time ago, Gelfand and Graev have constructed an action of the Weyl group on $D(G/U)$ by algebra automorphisms. The Gelfand-Graev construction was not algebraic, it involved the Hilbert space $L^2(G/U)$ in an essential way. We give a new algebraic construction of the Gelfand-Graev action, as well as its quasi-classical counterpart. Our approach is based on Hamiltonian reduction and involves the ring of Whittaker differential operators on G/U, a twisted analogue of $D(G/U)$. Our main theorem has an interpretation, via Geometric Satake, in terms of spherical perverse sheaves on the affine Grassmanian for the Langlands dual group.

Victor Guillemin
MIT
Quantizing G-actions on Poisson manifolds

In the first half of this talk I'll discuss some techniques for for dealing with this problem in general and in the second half describe some examples of applications of these techniques. (The results I'll be describing, by the way, are part of an ongoing collaboration with Eva Miranda and Jonathan Weitsman.)

Xuhua He
University of Maryland
Cocenters and representations of reductive p-adic groups

It is known that the number of conjugacy classes of a finite group equals the number of irreducible representations (over complex numbers). The conjugacy classes of a finite group give a natural basis of the cocenter of its group algebra. Thus the above equality can be reformulated as a duality between the cocenter (i.e. the group algebra modulo its commutator) and the finite dimensional representations.

Now let us move from the finite groups to the p-adic groups. In this case, one needs to replace the group algebra by the Hecke algebra. The problem is that the conjugacy classes are "mixed" together, which makes the cocenter (as well as representations) of p-adic groups more difficult to understand. The question is "can you hear every note from a musical chord?" Or in our situation, "can you separate the cocenter into nice subspaces?"

In this talk, I will explain the Newton decomposition of the cocenter and then some applications to the complex and modular representations of p-adic groups, including: a generalization of Howe's conjecture on twisted invariant distributions, trace Paley-Wiener theorem for smooth admissible representations, and the abstract Selberg Principle for projective representations. I will also talk about some connection to the orbital integrals.

Jing-Song Huang
HKUST
Dirac cohomology, orbit method and unipotent representations

The method of coadjoint orbits for real reductive groups is separated into three steps in correspondence with the Jordan decomposition of an orbit into hyperbolic part, elliptic part and nilpotent part (formulated in Vogan's 1986 ICM plenary speech). The hyperbolic step and elliptic step are well understood, while the nilpotent step to construct unipotent representations in correspondence with nilpotent orbits has been extensively studied in several different perspectives over the last thirty years. Nevertheless, the definition of unipotent representations remains to be finalized. The aim of this talk is to show that our recent work (joint with Pandzic and Vogan) on classifying unitary representations by their Dirac cohomology shed light on what kind of irreducible unitary representations should be defined as unipotent.

Anthony Joseph
Weizmann Institute
Adapted Pairs and polynomiality of invariants

Let $\mathfrak{a}$ be an algebraic Lie algebra and V a finite-dimensional $\mathfrak{a}$ module. An adapted pair (h, y) is a regular element $y \in V^*$ combined with a semisimple element $h \in \mathfrak{a}$ such that $hv = -v$. They can be hard to find but their construction is nevertheless just linear algebra. On the other hand the polynomiality of the invariant algebra $S(V)^{\mathfrak{a}}$ is a much harder matter. The relationship of these two questions will be reviewed as well as the many new examples which have been found.

Victor Kac
MIT
Integrable Hamiltonian partial differential and difference equations and related algebraic structures

The related algebraic structures are Lie pseudoalgebras, attached to a cocommutative Hopf algebra H (the case H=base field being Lie algebras). This turned out to be an adequate tool for the construction and study of integrable Hamiltonian partial differential and difference equations. The most famous of the former is the KdV equation, describing water waves in narrow channel. The most famous of the latter is the Volterra lattice, describing predator-prey interactions.

Some knowledge of Lie algebras, but no knowledge of integrable systems, will be assumed.

Masaki Kashiwara
RIMS
Quiver-Hecke algebras, quantum affine algebras and R-matrices

The module category of quiver Hecke algebras is a monoidal category whose Grothendieck algebra is isomorphic to the quantum unipotent coordinate ring. It is not a braid monoidal category, but there are R-matrices, which is very similar to the module category of a quantum affine algebra. This structure shows many properties of real simple objects.

This is a joint work with Seok-Jin Kang, Myungho Kim and Se-jin Oh.

Allen Knutson
Cornell University
Asymptotics of branching to symmetric subgroups

An early theorem of Bert describes the image of a coadjoint orbit $\mathcal{O}_\lambda$ of a compact group G under the torus moment map Φ_T, an asymptotic version of the question of which T-weights occur in the G-representation V_λ. The corresponding asymptotics of the weight *multiplicities* is $(\Phi_T)(dvol)$, the Duistermaat-Heckman measure. I'll give a positive formula, in terms of Littelmann-like paths, for the asymptotic of the K-multiplicities in a G-representation, where $K = G^\theta$ is a symmetric subgroup.

Toshiyuki Kobayashi
University of Tokyo
Branching problems and symmetry breaking operators

I plan to discuss some general results for restriction problems for reductive groups and explain the complete classification of symmetry breaking operators for differential forms on a pair of spheres. If time permits, I would like to discuss some relations to a conjecture of Gross and Prasad.

Shrawan Kumar
UNC Chapel Hill
Equivariant cohomology and K-theory of flag varieties

The talk will have two parts. The first part will be devoted to a review of some results of Kostant on the cohomology of nilradicals of parabolic subalgebras in a semisimple Lie algebra and the cohomology of flag varieties (including his (d, δ)-Hodge theory). The second part will be devoted to further developments in the area by among others: Kostant-Kumar, Arabia, Graham, Brion, Griffeth-Ram, Graham-Kumar, and Anderson-Griffeth-Miller. Many of the results also generalize to the Kac-Moody case but we will have no time to discuss them.

Ivan Losev

Northeastern University

On equivariantly irreducible modular irreducible representations of a semisimple Lie algebra

In this talk I will discuss the representation theory of semisimple Lie algebras $\mathfrak{g}$ in very large positive characteristic p. To an irreducible representation one can assign its p-character, an element of (the Frobenius twist) of $\mathfrak{g}^*$. The general case easily reduces to the situation when the p-character is nilpotent. While a lot is known about the irreducible representations and their classes in K_0, there is no combinatorial classification of the irreducibles and no explicit formulas for the K_0-classes for an arbitrary nilpotent p-character. A basic case creating difficulties is when the p-character is distinguished, i.e., is not contained in a proper Levi subalgebra. In my talk I will review some known results and then discuss my current work with Bezrukavnikov, where we get a combinatorial classification and Kazhdan-Lusztig type formulas for K_0-classes of equivariantly irreducible modules with distinguished p-character, where the equivariance is considered with respect to the action of the centralizer.

George Lusztig

MIT

Nilpotent orbits in semisimple Lie algebras and a cyclically graded analogue

I will survey the theory of nilpotent orbits including the early contribution of Bert Kostant. I will also talk about my recent joint work with Zhiwei Yun on nilpotent orbits associated to cyclically graded Lie algebras.

Anne Moreau

University of Lille

Quasi-lisse vertex algebras and chiral symlectic cores

In this talk, we will introduce the notion of chiral symplectic cores which can be viewed as chiral analogs of symplectic leaves. As an application we show that any quasi-lisse vertex algebra is a quantization of the arc space of its associated variety, in the sense that its reduced singular support coincides with the (reduced) arc space of its associated variety. Under reasonable assumptions, we conjecture that this associated variety is actually irreducible, and our result is a first step toward this conjecture. If times, I will also present an application to the vertex Poisson center of the coordinate ring of the arc space of Slodowy slices. All this is based on joint works with Tomoyuki Arakawa.

Paul-Emile Paradan
University of Montpellier
Symmetric pairs and branching laws

Let G be a complex reductive Lie group and let H be the subgroup fixed by an involution. A classical result assures that the H-action on the flag variety F of G admits a finite number of orbits. The purpose of this presentation is to explain how the irreducible representations (of finite dimension) of G, seen as H-modules, admit a finite decomposition parameterized by F/H.

Konstanze Rietsch
King's College London
Mirror symmetry for homogeneous spaces G/P

I will give an overview of results on mirror symmetry for G/P, with special emphasis on Grassmannians and Dubrovin/Givental style mirror symmetry.

Siddhartha Sahi
Rutgers University
The Capelli eigenvalue problem

In the 1980s Bert Kostant and I studied the eigenvalues of certain operators associated to Jordan algebras, which generalize the classical Capelli operator of invariant theory. Somewhat later, F. Knop and I discovered a remarkable connection between the spectrum of these operators and Macdonald polynomials, which led to advances in the theory of Cherednik algebras and to the proof of some conjectures of Macdonald.

I will describe this circle of ideas and also discuss some more recent developments, which have led to the resolution of questions of Howe-Lee and Shimura on invariant differential operators, and to related results for Lie superalgebras.

Eric Sommers
UMass Amherst
The defining equations for some nilpotent varieties

In his 1963 paper "Lie group representations on polynomial rings," Kostant found the defining equations for the nilpotent cone of a simple Lie algebra and also proved it is a normal variety. Later Broer showed uniformly that the closure of the next biggest nilpotent orbit, the subregular nilpotent orbit, is a normal variety and found its defining equations. We generalize Broer's technique to the class of nilpotent orbits that are Richardson orbits for orthogonal short simple roots. The proof involves cohomological results for line bundles on cotangent bundles of generalized flag varieties and a result related to flat bases of invariant polynomials. This is joint work with Ben Johnson.

Michele Vergne
University Denis Diderot, Paris
Moment map, generalized Horn inequalities and quivers

Work in common with Velleda Baldoni and Michael Walter.

Let G be a complex reductive group acting on a finite-dimensional complex vector space $\mathcal{H}$. Let $\mathfrak{t}^*_{\geq 0}$ be a choice of positive Weyl chamber. The moment map associated to a Hermitian metric on $\mathcal{H}$ determines a polyhedral cone $\mathrm{Cone}(\mathcal{H})$ in $\mathfrak{t}^*_{\geq 0}$. It is the cone generated by the T-weights of the polynomial functions on $\mathcal{H}$ which are semi-invariant under the Borel subgroup. We determine inductively the inequalities of the cone $\mathrm{Cone}(\mathcal{H})$ in the case of the linear representation of the group $G = \prod_x \mathrm{GL}(n_x)$ associated to a quiver and a dimension vector $n = (n_x)$ in terms of filtered dimension vectors. The inequalities obtained are generalizations of the Horn inequalities.

David Vogan
MIT
Quantization, the orbit method, and unitary representations

A characteristic of many of Kostant's papers is that the only formal prerequisite for reading them is linear algebra. I will try to meet that characteristic in this talk.

A unitary representation is a realization of a group as automorphisms of a Hilbert space. Since mathematical models of quantum mechanics live also on Hilbert spaces, it's natural to think of a unitary representation as a quantum-mechanical object. One can therefore ask what the corresponding classical object might be. In the 1960s Kirillov and Kostant proposed an answer to this question: the classical analog of a unitary [irreducible] representation is a symplectic manifold with a nice [transitive] group action. These classical analogues turn out to be easy to parametrize; the problem is a mild generalization of finding canonical forms for real matrices under conjugation.

Kirillov and Kostant suggested that these same geometric objects should also parametrize unitary representations. Fifty years later, we have not entirely understood how to implement the Kirillov-Kostant idea. I'll review some of what is known, and ideas for completing it.

Nolan Wallach
UC San Diego
My research work with Bertram Kostant

In this lecture I will describe my work with Bert over the period of over 10 years in which he visited UCSD for a month every Winter. I will discuss our published work on the Poisson version of Gelfand-Zeitlin theory, Maxwell's equations and representation theory, the variety of abelian Lie algebras and the structure of the singular set of a simple Lie algebra. I will also talk about some of the unpublished work.

Lauren Williams

UC Berkeley

Newton-Okounkov bodies for Grassmannians

In joint work with Konstanze Rietsch (arXiv:1712.00447) we use the X-cluster structure on the Grassmannian and the combinatorics of planar bicolored graphs to associate a Newton-Okounkov body to each X-cluster. This gives, for each X-cluster, a toric degeneration of the Grassmannian. We also describe the Newton-Okounkov bodies explicitly: we show that their facets can be read off from A-cluster expansions of the superpotential. And we give a combinatorial formula for the lattice points of the Newton-Okounkov bodies, which has a surprising interpretation in terms of quantum Schubert calculus.

Geordie Williamson

University of Sydney

Character formulas in the modular representation theory of algebraic groups

I will present two formulas for the characters of representations of algebraic groups in positive characteristic p. Both formulas involve p-Kazhdan-Lusztig polynomials and are expected to hold uniformly in the characteristic. (This is joint work with S. Riche, with parts also joint with P. Achar and S. Makisumi.)