

SYMPLECTIC GEOMETRY AND MICROLOCAL ANALYSIS:
IN HONOR OF PROFESSOR VICTOR GUILLEMIN ON THE OCCASION OF HIS
80TH BIRTHDAY

FRIDAY, NOV. 10

Gunther Uhlmann

(HKUST and Univ. of Washington)

Can you hear the shape of a space or the inner structure of the Earth?

Alan Weinstein

(Berkeley)

Hamiltonian Lie algebroids and general relativity

In general relativity, the gravitational field is a lorentzian metric on a 4-dimensional space-time manifold. The Einstein field equations may be expressed, at least locally in time, as the trajectories of a hamiltonian system on a cotangent bundle T^*R , where R is the (infinite dimensional) manifold R of riemannian metrics on a 3-dimensional "time slice", with initial values constrained to a certain submanifold C of T^*R .

Geometric properties of C suggest that the constraints should be related to the symmetry group of the Einstein equations, consisting of the diffeomorphisms of space-time. But this group does not act on R , since it does not act on an individual time slice. Blohmann, Fernandes, and the speaker have shown that the algebraic structure of the constraints is in fact related to a *groupoid* of diffeomorphisms between pairs of time slices, but a direct connection between the constraints and this groupoid was not found.

I will report on ongoing work with Blohmann and Schiavina establishing this direct connection, centered around an extension to Lie algebroids (the infinitesimal version of Lie groupoids) of the theory of hamiltonian actions of Lie algebras. Aside from the application to relativity, the theory leads to interesting questions in symplectic topology.

Ana Cannas de Silva
(ETH)

Embedded Lagrangians in CP^2

The central question “what lagrangians are there in a given symplectic manifold?” comes in different flavours depending on further desired properties. We concentrate on embedded (closed) lagrangians in CP^2 that sit nicely with respect to the toric structure and discuss an example that exhibits a distinguishing behavior under reduction relevant in connection with Weinstein’s lagrangian composition and work of Wehrheim and Woodward in Floer theory.

Eckhard Meinrenken
(Toronto)

Euler-like vector fields

A vector field is “Euler-like” with respect to a submanifold if its linear approximation is the Euler vector field on the normal bundle. We show that such a vector field canonically determines a tubular neighborhood embedding, and use this fact to discuss various ‘normal form theorems’ in differential geometry.

SATURDAY, NOV. 11

Maciej Zworski
(Berkeley)

Magnetic oscillations in a model of graphene

About 25 years ago Victor Guillemin and I ran a seminar on Helffer–Sjostrand’s approach to magnetic oscillations in three dimensional crystals (the de Haas–van Alphen effect). I will explain how what I learned back then can be used to describe magnetic oscillations in the simplest mathematical model for graphene given by quantum graphs. Using semiclassical methods (with the strength of the magnetic field as the small parameter) we obtain a geometric description of the density of states showing asymmetry seen in physical experiments but not in commonly used perfect cone approximations. Joint work with S Becker.

John Toth
(McGill)

Some recent results on asymptotic eigenfunction bounds

My talk will be a survey of some recent results on improvements in L^∞ bounds for Laplace eigenfunctions. I will also discuss recent work on asymptotic lower bounds for eigenfunction restrictions to hypersurfaces and applications to the study of nodal sets.

Leonid Friedlander
(Arizona)

On multiplicative properties of determinants

Let an elliptic operator of positive order be factored as $A(I + T)$ where T is an operator of negative order that does not belong to the trace class. I relate the determinants of that operator to the determinant of A and regularized Fredholm determinant of $I + T$.

Gerardo Mendoza
(Temple)

Elliptic operators on compact manifolds with simple strata

I'll give an overview of results concerning several aspects of the analysis of elliptic operators on compact stratified manifolds with a single stratum. In the realm of conical singularities I'll discuss the general asymptotics of the resolvent of an elliptic operator assuming only existence of rays of minimal for the principal symbols, and recent results on the nature of domains of elliptic complexes. In connection with boundary value problems for higher dimensional strata, I'll describe the bundle of traces (the bundle of Cauchy data), some aspects of the analysis on these, and a result on the domain of the Friedrichs extension (from its minimal domain) of an elliptic semi-bounded second order operator. The results to be presented were obtained in collaboration with Juan Gil or Thomas Krainer or both.

Allen Knutson
(Cornell)

Geometry of the individual terms in the DH formula

Guillemin, Lerman, and Sternberg gave a polyhedral interpretation of the Duistermaat-Heckman formula for their measure, an alternating sum of projections of cones. I'll explain how the theory of Chern-Schwarz-MacPherson classes gives a geometric interpretation of the individual terms; they are themselves DH measures of certain cycles in the cotangent bundle. In the case of coadjoint orbits of a compact group, these cycles have seen a lot of interest recently (in e.g. the works of Maulik-Okounkov, Aluffi-Mihalcea, Huh...), where they give a deformation of the Schubert basis of homology. I'll give a formula for their coproduct structure constants, in the case of Grassmannians (this work joint with P. Zinn-Justin).

Victor Guillemin
(MIT)

Torus actions with bi-collinear weights

This talk is about some generalizations that Sue Tolman, Catalin Zara and I have been investigating of the Goresky-Kottwitz-MacPherson theorem for torus actions with non-collinear weights.

SUNDAY, NOV. 12

Rafe Mazzeo
(Stanford)

The large scale structure of the Hitchin moduli space

I will report on a set of several interconnected works to describe asymptotic features of the moduli space of Higgs bundles on a Riemann surface. This includes progress toward a conjecture of Gaiotto-Moore-Neitzke about the nature of the hyperKähler metric, some local and global aspects of the ABA brane deformation theory in this moduli space, and some interesting explicit examples.

Dan Burns
(Michigan)

Twistor transforms and geodesics, real and complex

In the mid-1970's, Victor Guillemin settled the conjecture of Funk on the structure of the space of Zoll metrics on the sphere S^2 near the round metric. His analysis centered on the Radon transform associated to such a metric. LeBrun-Mason gave a new proof of this much later based on a kind of two-dimensional twistor transform coming from the set of almost complex structures on S^2 . They also extended their techniques to the case of self-dual metrics of signature (2,2) on $S^2 \times S^2$ satisfying a "Zollfrei" condition (Guillemin) analogous to the Zoll condition, but for null geodesics.

We report on an ongoing project with John Bland and Kin-Kwan Leung re-examining LeBrun-Mason's four-dimensional theory, but with differing underlying model spaces, and for anti-self dual metrics, whose geometry is surprisingly different. The main observation is that for a smooth, strictly convex domain $D \subset \mathbb{C}^2$ the moduli space of Kobayashi disks in D , the holomorphic geodesics for the Kobayashi metric, is a four dimensional, anti-self-dual manifold of signature (2,2). We determine the twistor space for this manifold, and show how to recover the Kobayashi disks from it. The resulting duality is as precise as the duality between geodesics and points on S^2 in the original Zoll case. Some parts of this project are fully proven, all are proven for the model case $D = \mathbb{B}^2$, the unit ball. Many open questions remain.

Tara Holm
(Cornell)

Fundamental groups of toric origami manifolds

A folded symplectic form on a manifold is a closed 2-form with the mildest possible degeneracy along a hypersurface. A special class of folded symplectic manifolds are the origami manifolds. In the classical case, toric symplectic manifolds can be classified by their moment polytope, and their topology (equivariant cohomology) can be read directly from the polytope. In this talk we examine the toric origami case: we will describe how toric origami manifolds can also be classified by their combinatorial moment data, and present some results about the topology of toric origami manifolds. Feedback on the "right" combinatorial, geometric and topological questions to ask will be very much appreciated. This is based on joint work with Ana Rita Pires and further work by my PhD student Ian Pendleton.

Alejandro Uribe
(Michigan)

Spectral results for singular Toeplitz operators in Bargmann spaces

I will consider Toeplitz operators in Bargmann spaces whose multiplier is a delta function supported on an isotropic (or Lagrangian) submanifold times a smooth density. These are trace-class operators that arise naturally as “mixed states” quantizing the submanifold, without the need to assume a Bohr-Sommerfeld condition. The main result is a non-standard Szego limit theorem. Other results include estimates of the largest eigenvalue, and asymptotics of Schatten norms. This is joint work with Salvador Pérez-Esteva (UNAM). I will also discuss how these results should be obtainable via a symbol calculus of Hermite states in complex polarizations (work in progress).

Lisa Jeffrey
(Toronto)

The triple reduced product and Higgs bundles

We identify the symplectic quotient of the product of three coadjoint orbits in $SU(3)$ with a space of Higgs bundles over the 2-sphere with three marked points, where the residues of the Higgs fields at the marked points are constrained to lie in the three coadjoint orbits. By considering the spectral curves for the Hitchin system, we identify the moment map for a Hamiltonian circle action on the reduced product. We use the earlier work of Adams, Harnad and Hurtubise to find Darboux coordinates.