2021 SPUR (Summer Program in Undergraduate Research) Conference

Friday, August 06, 2021

Zoom webinar

9:30am: Conference Opening by SPUR faculty advisors Prof. David Jerison and Prof. Ankur Moitra

9:35am: Dain Kim and Anqi Li, "Cubic Goldreich-Levin" (mentor: Jonathan Tidor)

10:05am: Preston Cranford and Peter Rowley, "Finding bounded simplicial sets with finite homology" (mentor: Robert Burklund)

10:45am: Enrico Colon, "Split-multiplicity-free flagged Schur polynomials" (mentor: Yibo Gao)

11:15am: Frank Wang, "Toward explicit Hilbert series of quasi-invariant polynomials in characteristic p" (mentor: Calder Morton-Ferguson)

11:45am: Yuyuan Luo, "Explicit Construction of the Hecke Character Associated to Certain CM Elliptic Curves" (mentor: Gefei Dang)

1:00pm: Amanda Ledesma Vanegas, "A reduced Swift-Hohenberg style model for vegetation patterns in drylands" (mentors: Davis Evans and Alasdair Hastewell)

1:30pm: Alan Peng, "Convolution-exact perverse sheaves on the affine flag variety" (mentor: Calder Morton-Ferguson)

2:15pm: Daishi Kiyohara, "A new approach to the upper estimate of lattice points on a curve via ℓ^2 decoupling" (mentor: Feng Gui)

2:45pm: Jacob Lerma and Jeffery Opoku-Mensah, "Large positivity of the mass using $p-$harmonic functions on asymptotically flat 3-manifolds" (mentor: Julius Baldauf)

3:15pm: Conference Closing

Dain Kim and Anqi Li

Cubic Goldreich-Levin

Mentor: Jonathan Tidor

Project suggested by Jonathan Tidor

We establish a cubic Goldreich-Levin algorithm which makes polynomially-many queries to a function $f\colon \mathbb{F}_p^n \to \mathbb{C}$ and produces a decomposition of f as a sum of cubic phases and a small error term. This is a natural higher order generalization of the classical Goldreich-Levin algorithm. The classical (linear) Goldreich-Levin algorithm has wide-ranging applications in learning theory, coding theory and the construction of pseudo-random generators in cryptography, as well as being closely related to Fourier analysis. Higher order Goldreich-Levin algorithms on the other hand involve central problems in higher order Fourier analysis, namely the inverse theory of the Gowers U^k norms, which are well-studied in additive combinatorics. The only known result in this direction prior to this work is the quadratic Goldreich-Levin theorem, which was proved by Tulsiani and Wolf in 2011. The main step of their result involves an algorithmic version of the U^3 inverse theorem. More complications appear in the inverse theory of the U^4 and higher norms. Our cubic Goldreich-Levin algorithm is based on algorithmizing recent work by Gowers and Milićević who proved new quantitative bounds for the U^4 inverse theorem. In this talk, we will share our new results on the cubic Goldreich-Levin algorithm and briefly discuss some extensions we have worked on.

Preston Cranford and Peter Rowley

Finding Bounded Simplicial Sets with Finite Homology

Mentor: Robert Burklund

Project suggested by Robert Burklund

A central problem in computational algebraic topology surrounds the computation of the homotopy groups of a given space, represented as a simplicial set. Algorithms have been derived which achieve this, but the running times depend on the size of the input simplicial set. In order to reduce this dependence on the simplicial set chosen, we describe in this paper a procedure which, given a finite, simply-connected simplicial set of dimension d with finite integral homology, finds a simplicial set with size upper bounded by a function of homology and d. This implies that a simply-connected finite homotopy type with finite integral homology has a finite simplicial set representative whose size is bounded by a function of its homology.

Enrico Colón

Split-multiplicity-free flagged Schur polynomials

Mentor: Yibo Gao

Project suggested by Yibo Gao

Quiver coefficients come from the study of a general kind of degeneracy locus associated to an oriented quiver of type A. They can be obtained by expanding the Schubert polynomials into the split-Schur polynomials, and possess very rich combinatorial structures. In this paper, we investigate the problem of determining which Schubert polynomials are split-multiplicity-free by looking at two meaningful special cases: the flagged Schur polynomials, which are Schubert polynomials of vexillary permutations, and the Stanley symmetric polynomials, which are stable limits of Schubert polynomials. Specifically, we present a necessary and sufficient condition on a shape λ for the flagged Schur polynomials s_λ^b to be split-multiplicity-free, given a generic flag. We also discuss progress on the Stanley symmetric polynomials via Rothe diagram.

Frank Wang

Toward explicit Hilbert series of quasi-invariant polynomials in characteristic

Mentor: Calder Morton-Ferguson

Project suggested by Profs. Roman Bezrukavnikov and Pavel Etingof

We study the spaces Q_m of m-quasi-invariant polynomials of the symmetric group S_n in characteristic p. Using the representation theory of the symmetric group we describe the Hilbert series of Q_m for $n = 3$, proving a conjecture of Ren and Xu. From this we may deduce the palindromicity and highest term of the Hilbert polynomial and the freeness of Q_m as a module over the ring of symmetric polynomials, which are conjectured for general n. We also prove further results in the case $n = 3$ that allow us to compute values of m, p for which Q_m has a different Hilbert series over characteristic 0 and characteristic p, and what the degrees of the generators of Q_m are in such cases. These latter results allow us to check the validity of another conjecture in of Ren and Xu for a much larger range of values than were previously verified.

Yuyuan Luo

Explicit Construction of the Hecke Character Associated to Certain CM Elliptic Curves

Mentor: Gefei Dang

Project suggested by Prof. Wei Zhang

Deuring proved that each CM elliptic curve have a corresponding Hecke character with the same L-function, hence showing that the L-functions of CM elliptic curves have analytic continuation in $\mathbb{C}$. In this paper, we explicitly construct the Hecke character for a particular elliptic curve whose endomorphism ring is an order of $\mathbb{Q}(\sqrt{-7})$, with views towards generalizing this result to classes of CM elliptic curves.

Amanda Ledesma Vanegas

A Reduced Swift-Hohenberg Style Model for Vegetation Patterns in Drylands

Mentors: Davis Evans and Alasdair Hastewell

Project suggested by Profs. John Bush and Jörn Dunkel

One in three people worldwide live in drylands, which are particularly vulnerable to desertification. In these ecosystems, where natural resources are limited, vegetation survives by organizing itself into patterns. The ecological factors governing these patterns were captured by Zelnik and others in a reaction-diffusion model consisting of two equations for groundwater and biomass. In this paper, we leverage the strong water-biomass relation suggested by our simulations to derive a groundwater equation in terms of biomass and its spatial derivatives. Through this procedure we reduce Zelnik's model to a single Swift-Hohenberg style biomass equation. Our model differs from the classical Swift-Hohenberg models in that nonlinear terms multiply gradients of biomass. This feature reflects that vegetation regulates its own water induced growth. The model presented shares the pattern forming properties of Swift-Hohenberg models, making it mathematically tractable while conserving clear ecological meaning. Hence, our biomass equation allows for the extensive study of drylands' resilience to changing climate conditions.

Alan Peng

Convolution-exact perverse sheaves on the affine flag variety

Mentor: Calder Morton-Ferguson

Project suggested by Prof. Roman Bezrukavnikov

An unpublished result by Mirković states that convolution-exact sheaves on the flag variety of a simple linear algebraic group G over $\overline{\mathbb{F}_p}$ are tilting, that is, they admit both standard and costandard filtrations. The analogous statement for convolution-exact sheaves on the affine flag variety is false, but Arkhipov and Bezrukavnikov noted that it is still not known whether the projections of such sheaves to a different category, which they called the Iwahori–Whittaker category, are tilting. We make partial progress toward this question by considering reductions to the combinatorics of the extended affine Weyl group of G. In particular, we demonstrate an obstruction to a direct generalization of Mirković's original proof, even in the $G = \mathrm{SL}_2$ case. We also investigate Wakimoto filtrations, as introduced by Arkhipov and Bezrukavnikov, and their variants.

Daishi Kiyohara

A new approach to the upper estimate of lattice points on a curve via ℓ^2 decoupling

Mentor: Feng Gui

Project suggested by Prof. Larry Guth

In this paper we examine the number of $\frac{1}{N}$-integral points on a fixed planar curve Γ. We prove that, if the curve Γ is $C^{n,\alpha}$ for some $\alpha > 0$ and satisfies a certain analytic condition, then we have $|\Gamma \cap (\frac{1}{N}\mathbb{Z})^2| \lesssim N^{e(n)+\epsilon}$ for any $\epsilon > 0$ with a function $e(n)$ which is asymptotically $n^{-1/2}$, where the implicit constant depends on ϵ. This gives an extension of Bombieri and Pila's result to curves of any regularity. Our approach is to lift up a planar curve into higher dimensional spaces and apply ℓ^2 decoupling inequalities there.

Jacob Lerma and Jeffery Opoku-Mensah

Positivity of the mass using $p-$harmonic functions on asymptotically flat 3-manifolds

Mentor: Julius Baldauf

Project suggested by Julius Baldauf

The positive mass theorem states that for asymptotically flat spacetimes, the ADM mass, defined by Arnowitt-Deser-Misner, is nonnegative. This theorem has been given several proofs, notably by Schoen-Yau, Witten, Geroch, and others. We generalize an argument of Jang which proves the positivity of the ADM mass of an asymptotically flat spacelike hypersurface (M^3, g) with a single exterior region $M_{ext} \cong \mathbb{R}^3 \setminus B_1(0)$. We focus on a quantity asymptotic to the Hawking quasi-local mass defined on level sets of a p-harmonic function, for $1 < p < 3$, and demonstrate a monoticity formula similar to the Geroch monotonicity formula. For $p = 2$, we recover Jang's argument.