

Anay Aggarwal

Weak Error Analysis and Small-Noise Optimization of
Stochastic Linear Multistep Methods (SLMMs) for Itô
and Lévy SDEs

under the direction of Joonsoo Lee

Abstract

We study the weak convergence behavior of stochastic linear multistep methods (SLMMs) for solving stochastic differential equations (SDEs). While SLMMs have been previously investigated in terms of strong error, their weak convergence properties remain largely unexplored. We develop a general framework for computing the full weak error expansion of arbitrary SLMMs using the Kolmogorov backward equation and linear recurrence techniques. This framework enables a systematic analysis of both existing and newly proposed multistep methods. In particular, we construct SLMMs that achieve improved weak accuracy in small noise regimes. We extend our analysis to Lévy-driven SDEs and derive precise weak error bounds for a class of multistep schemes that interpolate drift and diffusion terms. Numerical simulations confirm the theoretical predictions and illustrate regimes in which multistep methods significantly outperform standard one-step approaches in weak approximation accuracy.

Shruti Arun

On q -Analogues of the Markov Equation

under the direction of Mohit Hulse

Abstract

Exceptional vector bundles over $\mathbb{P}_{\mathbb{C}}^2$ correspond to solutions to the Markov equation. We are interested in analogous objects over $D_0^b \text{Coh } \mathbb{K}\mathbb{P}_{\mathbb{C}}^2$ and $D_0^b \text{Coh } \mathbb{T}^*\mathbb{P}_{\mathbb{C}}^2$. Specifically, we focus on collections of coherent sheaves that correspond to simple modules over a Koszul ring A under the equivalences $D_0^b \text{Coh } \mathbb{K}\mathbb{P}_{\mathbb{C}}^2 \simeq D^b \text{Mod } A$ and $D_0^b \text{Coh } \mathbb{T}^*\mathbb{P}_{\mathbb{C}}^2 \simeq D^b \text{Mod } A$. Under their K -theory, these interesting collections correspond to Koszul bases in $\mathbb{Z}[t^{\pm}]^{\oplus 3}$, which we study. We have results on the structure of these Koszul bases in the case of $\mathbb{K}\mathbb{P}_{\mathbb{C}}^2$ and a closed form for a family of such Koszul bases in $\mathbb{T}^*\mathbb{P}_{\mathbb{C}}^2$. We also describe an explicit algorithm taking these collections of coherent sheaves over $\mathbb{K}\mathbb{P}_{\mathbb{C}}^2$ to exceptional vector bundles over $\mathbb{P}_{\mathbb{C}}^2$.

Rachel Chen

Explicit Irreducible Decomposition of the Spin Representation of the Temperley-Lieb Algebra

under the direction of Kenta Suzuki

Abstract

The spin representation $(\mathbb{C}^2)^{\otimes n}$ can be decomposed uniquely into irreducible modules called the Specht Modules, denoted W_k^n . Lusztig's dual canonical basis of the spin representation can be viewed as diagrams of the Temperley-Lieb algebra, and the Specht Modules also have a diagrammatic basis. We explicitly describe the decomposition of the spin representation into Specht Modules, $(\mathbb{C}^2)^{\otimes n-2} \cong W_{n-2}^{n-2} \oplus \cdots \oplus W_{n-2k}^{n-2}$, by computing the images of the diagrammatic basis elements of the Specht Modules. We do this by using induction to compute the image of W_n^n and reducing the problem of computing the images of other Specht modules to this case. Our results may lead to the notion of a canonical basis for Specht Modules in the future.

Samanyu Ganesh

Characterizing Variations in the Ruelle Zeta Function
Under Conformal Perturbations to the Metrics of
Negatively Curved Riemannian 2-Manifolds

under the direction of Alain Kangabire

Abstract

The Ruelle zeta function $\zeta_R(s)$ is a dynamical analog of the Riemann zeta function, defined via the lengths of primitive closed geodesics on a compact Riemannian surface. For negatively curved surfaces, the flow is chaotic and the set of closed orbits is discrete and stable, making $\zeta_R(s)$ well-suited for spectral analysis. In particular, the vanishing order of $\zeta_R(s)$ at $s = 0$ is determined by the Euler characteristic, and its leading coefficient is conjecturally linked to analytic torsion, a spectral invariant of the Laplacian on manifolds. In this paper, we study how that leading coefficient varies under conformal perturbations of the hyperbolic metric of the form $g_\varepsilon = e^{\varepsilon\phi}g_{\mathbb{H}}$. Using recent microlocal techniques developed for resonances of Anosov flows, we reduce the problem to solving the inhomogeneous transport equation $(X + 1)(U_- \dot{f}_0) = h$ on the unit cosphere bundle. With explicit expressions for crucial Fourier modes of $\dot{f}_0$ (-1 and 1 , specifically), we compute the variation of the zeta function up to second order in ε , offering evidence for the Fried conjecture in new cases.

Sophia Jin

Coloring-Based Knot Invariants in Projective Space

under the direction of Kenta Suzuki

Abstract

In this paper, we will determine and characterize knot invariants related to coloring in $\mathbb{RP}^3$ and $\mathbb{RP}^2 \times (0, 1)$. In $\mathbb{S}^3$, there exist notions of colorings related to coloring the regions and strands, and we extend those to $\mathbb{RP}^3$ and $\mathbb{RP}^2 \times (0, 1)$. In $\mathbb{RP}^2 \times (0, 1)$, we can establish an involutive function that determines the colors of antipodal points and attach it to the structure of a quandle. We attach such a function to the Joyce quandle, giving us a knot invariant that allows us to distinguish between two links. However, such a function does not work as well in $\mathbb{RP}^3$, so we instead establish a knot invariant from coloring regions of the projections of certain links, based on the Dehn presentation of the fundamental group of the knot complement. We establish a similar invariant for all links in $\mathbb{RP}^2 \times (0, 1)$.

Jaeho Lee

Quadratic Large Sieves for “Algebraically Impossible”
Sets

under the direction of Samuel Packman

Abstract

Linnik’s large sieve provides a bound for the sifted set X constructed by removing a positive proportion of residue classes modulo p . We consider quadratic sieves that classify quadratic residues and take Boolean variations of that sieve. We show that the number of $n \leq N$ such that n and $n + k$ are quadratic residues modulo p for $N^{1/m} < p < 2N^{1/m}$ for some positive integer m is bounded by $(\log N)^{4m+1}$. We then work with the case where at least one of $n + k_i$ for some set of constants k_i is a quadratic residue modulo p for all $N^{1/m} < p < 2N^{1/m}$, showing that the asymptotic bound is identical to the case where n must be a quadratic residue.

Shuxuan (Judy) Li

Hilbert Series of Quasi-invariant Polynomials of D_n in
Characteristic p

under the direction of Frank Wang

Abstract

Let $Q_m(W, \mathbb{F})$ denote the space of m -quasi-invariant polynomials of a finite Coxeter group W over a field $\mathbb{F}$. The Hilbert series of both $Q_m(S_n, \mathbb{C})$ and $Q_m(S_n, \mathbb{F}_p)$ have already been extensively studied. Quasi-invariant polynomials of the dihedral group have only been studied by Feigin and Veselov in 2003, who proved the general form of the Hilbert series of $Q_m(D_n, \mathbb{C})$. In this paper, we explore the Hilbert series of $Q_m(D_n, \mathbb{F}_{p^k})$, where k is such that a primitive $2n^{\text{th}}$ root of unity is defined in $\mathbb{F}_{p^k}$. In particular, we prove a sufficient condition for which the Hilbert series of $Q_m(D_n, \mathbb{F}_{p^k})$ is different from that of $Q_m(D_n, \mathbb{C})$. In addition, we prove results about the generators of $Q_m(D_n, \mathbb{F}_{p^k})$, which are relevant to our conjectured form of the Hilbert series of $Q_m(D_4, \mathbb{F}_{p^k})$. In doing so, we come closer to understanding quasi-invariant polynomials with respect to any finite Coxeter group.

Aaron Lin

Labeled Chip-Firing on k -ary and 3-regular Trees

under the direction of Ryota Inagaki

Abstract

We explore labeled and unlabeled chip-firing on 3-regular and k -ary trees, trees where every vertex has degree 3 and k , respectively. First, we extend known results for binary trees such as the endgame and the zigzag bound to k -ary trees. Next, we use the endgame to prove various properties of stable configurations, including zigzag relation between the chip on a vertex and the chip on its parent and the location of the smallest and largest chips in a subtree. We then use these properties to construct a recursive upper bound, known as the *zigzag bound*, on the number of stable configurations in labeled chip-firing on k -ary trees. This generalizes previously known bounds for binary trees to k -ary trees in general. We further provide a novel lower bound through a unique recursive construction of stable configurations. Lastly, we provide results on unlabeled chip-firing on 3-regular trees and show that in the labeled case, the smallest and largest chip can lie anywhere on the leftmost and rightmost diagonal, respectively, in order to construct an upper bound on the number of stable configurations.

Susie Lu

On the Maximum Spread of Directed Graphs

under the direction of Prof. John Urschel and Genaro Laymuns

Abstract

For a directed graph G , its spread is the largest distance between any two eigenvalues of its adjacency matrix in the complex plane. In 2022, Breen, Riasanovsky, Tait, and Urschel posed the open problem of finding the maximum spread among all n -vertex directed graphs, allowing self-loops. In this paper, we resolve this open problem. We prove that the spread of any n -vertex directed graph is at most $2n/\sqrt{3}$, and this bound is tight up to an additive factor. In particular, when n is a multiple of 3, the maximum spread is exactly $2n/\sqrt{3}$. When n is not a multiple of 3, the maximum spread is at least $2n/\sqrt{3} - 1/(2\sqrt{3}n)$. Further, our results imply that the maximum spread graph among n -vertex directed graphs is the same as the maximum spread graph among n -vertex undirected graphs. In other words, allowing directed edges does not increase the largest spread that can be achieved.

Dimana Pramatarova

Investigating the Periodicity of Weighted Catalan
Numbers and Generalizing Them to Higher Dimensions

under the direction of Ryota Inagaki

Abstract

The weighted Catalan numbers, like the Catalan numbers, have various combinatorial interpretations, such as the number of *weighted Dyck paths*. These are paths in $\mathbb{Z}^2$, composed of up-steps of $(1, 1)$ and down-steps of $(1, -1)$, starting from $(0, 0)$ and ending at $(2n, 0)$, that never go below the x -axis and have a certain weight function. We investigate the periodicity of these sequences modulo m . First, we consider a notable subcase, namely the *bounded* Catalan numbers, and obtain results on their period modulo 2^r , where r is a positive integer. Further, we define a weighted analogue for the multidimensional Catalan numbers, obtain matrix-based recurrences for some of them, and give conditions under which they are periodic. Building on this framework, we introduce two new sequences of triangular arrays: the first one enumerates the k -dimensional Dyck paths of exact height s ; the second one is a new multidimensional generalization of the Narayana numbers, which count the number of Dyck paths with exactly p peaks.

Allen Qian

Parameter Estimation in Dynamic Bayesian Networks

under the direction of Joonsoo Lee

Abstract

Dynamic Bayesian Networks (DBNs) have received much attention over the past few decades, with potential application in fields like medicine for the inference of conditions, environmental modeling for risk assessment, machine learning, and many other fields. However, the problem of parameter estimation within these networks remains an open problem. This paper focuses on this problem, particularly the weights defining node interactions over time. We derive Maximum Likelihood Estimators (MLEs) for the linear Gaussian case, proving that the optimal weights satisfy a system of equations involving empirical moments. We further establish non-asymptotic bounds on estimation error and prove convergence at the rate of $O(\frac{1}{\sqrt{N}})$ under sub-Gaussian assumptions. We develop a general framework for nonlinear update rules, showing that parameter estimates can be obtained for any general update function. Simulations validate these theoretical findings and show how graph structure and sample size affect estimation accuracy. Our results provide insight into the role of network structure in parameter learning and point toward future directions for extending these methods to nonlinear or non-Gaussian settings.