

# Clique Number Of Random Cayley Graphs

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Abstract: Take a finite abelian group  $G$ , and a subset  $A$  of  $G$ . One can form a graph, the Cayley graph of  $A$ , on vertex set  $G$  by joining  $x$  to  $y$  iff  $x + y$  lies in  $A$ . The Paley sum graphs (in which  $G = \mathbb{Z}/p\mathbb{Z}$  and  $A$  is the set of quadratic residues) are examples of graphs of this type, and they are often supposed to share many of the properties of random graphs.

We investigate the properties of random Cayley graphs, in which  $A$  is chosen randomly (each element  $x$  in  $G$  being chosen independently with probability  $1/2$ ). In particular we look at the clique number of a random Cayley graph. When  $G = \mathbb{Z}/p\mathbb{Z}$  the clique number is almost surely  $O(\log |G|)$ , the same order of magnitude as a true random graph. However when  $G = (\mathbb{Z}/2\mathbb{Z})^m$  the clique number is almost surely comparable to  $\log |G| \log \log |G|$ , which is of course rather larger.

It turns out that this is really a problem in additive number theory. We will discuss why this is so, and in particular we will talk about Freiman's theorem on sets with small sumset and the role it has to play here.