Problem 1. Let $X$ be a set, and let $A, B \subset X$ be two subsets. Write down a rigorous proof of the equalities

$$(A \cup B)^c = (A^c) \cap (B^c)$$

and

$$(A \cap B)^c = (A^c) \cup (B^c).$$

Hint: Use the style of proof which we saw on the example sheet. Each set equality consists of two inclusions.

To give a starting point, the argument for the inclusion $(A \cup B)^c \subset (A^c) \cap (B^c)$ might begin with: “Let $x \in (A \cup B)^c$ be given. We then have $x \in X$, but $x$ does not belong to the set $A \cup B$ (that is, the statement “$x \in A$ or $x \in B$” is false). (...)”