

Introduction to Proofs
IAP 2015
In-class problems for day 10

Problem 16. Use the Cauchy-Schwarz inequality to prove that for all $a, b, c \in \mathbb{R}$,
$$a^2 + b^2 + c^2 \geq ab + bc + ca.$$

Proof.

□

Problem 17. Fix $n \in \mathbb{N} \setminus \{0\}$. Use the Cauchy-Schwarz inequality to prove that if a_1, a_2, \dots, a_n are a given collection of n real numbers, then the inequality

$$\sum_{i=1}^n \frac{a_i^2}{h_i} \geq \frac{(\sum_{i=1}^n a_i)^2}{\sum_{i=1}^n h_i}$$

holds for all $h_1, \dots, h_n \in \mathbb{R}$ with each $h_i > 0$.

Proof.

□