

18.S097 Introduction to Proofs  
IAP 2015  
Homework 4  
Due: Wednesday, Jan. 14, 2015

**Problem 1.** Define the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = x \quad \text{for } x \in \mathbb{Q}$$

and

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R} \setminus \mathbb{Q}.$$

Show that  $f$  is continuous at 1 and is not continuous at 2.

Hint: Recall that the statement that  $f$  is not continuous at 2 means (since it is the negation of the statement that  $f$  is continuous at 2) that the following condition is satisfied: there exists  $\epsilon > 0$  such that, for all  $\delta > 0$ , there exists  $x \in \mathbb{R}$  with  $|x - 2| < \delta$  and  $|f(x) - f(2)| = |f(x) - 2| > \epsilon$ .