

18.S097 Introduction to Proofs

IAP 2015

Homework 3

Due: Monday, Jan. 12, 2015

Problem 1. *Let A and B be arbitrarily given sets.*

- (1) *Show that a function $f : A \rightarrow B$ is injective if and only if there exists a left inverse $g : B \rightarrow A$ for f in the sense that $g(f(x)) = x$ for all $x \in A$.*

Similarly, given two functions $f : A \rightarrow B$ and $g : B \rightarrow A$, we say that g is a right inverse for f if $f(g(x)) = x$ for all $x \in B$.

Establish the following claims:

- (2) *A given function $f : A \rightarrow B$ is surjective if and only if there exists a right inverse $g : B \rightarrow A$ for f .*
- (3) *Suppose that $f : A \rightarrow B$ has both a left inverse $g_1 : B \rightarrow A$ and a right inverse $g_2 : B \rightarrow A$. Then $g_1 = g_2$ (that is, $g_1(x) = g_2(x)$ for every $x \in B$).*