18.906: Problem Set IV

Due Monday, May 8, in class.

15. **(a)** Obtain the lower bounds on codimensions immersions of $\mathbb{R}P^n$ into Euclidean spaces which are implied by Stiefel-Whitney classes.

(b) Write n as a sum of distinct powers of 2: $n = 2^{k_1} + 2^{k_2} + \cdots$. What to the Stiefel-Whitney classes tell you about the minimal codimension of an immersion of $\mathbb{R}P^{2^{k_1}} \times \mathbb{R}P^{2^{k_2}} \times \cdots$ into Euclidean space?

16. **(a)** Since all integral characteristic classes of complex vector bundles are polynomials in Chern classes, the Euler class (of the underlying oriented real vector bundle) must have such an expression. What is it?

Similarly, all mod 2 characteristic classes of real vector bundles are polynomials in Stiefel-Whitney classes, so the Euler class must have such an expression. What is it?

(b) Since all mod 2 characteristic classes of complex vector bundles are polynomials in Chern classes reduced mod 2, the Stiefel-Whitney classes of the underlying real vector bundle must have such expressions. What are they?

17. Express the Chern classes of $E_1 \otimes E_2$ in terms of the Chern classes of E_1 and E_2, in the following three cases: $\dim E_1 = \dim E_2 = 1$; $\dim E_1 = 1$ and $\dim E_2 = 2$; $\dim E_1 = \dim E_2 = 2$.

18. Recall that a space has "category $\leq k$" if it admits a covering by $k+1$ opens A_i such that $A_i \to X$ is null-homotopic for each i. The ring structure of the cohomology of a space X determines a lower bound on the "category" of X. What is that bound for the complex Grassmannian $\mathrm{Gr}_2(\mathbb{C}^4)$?

19. Put your formula for the tangent bundle of a Grassmannian (which gave it to you as a complex vector bundle) to work to determine the Chern classes of $\mathrm{Gr}_2(\mathbb{C}^4)$.

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