In this short note we prove a simple lemma that fills in a gap (and corrects a typo) at then end of the proof of [1, Lemma 4.6.3].

**Lemma.** Let $f \in M_k(N, \chi)$ with $k > 0$, let $\ell > 1$ be an integer prime to $N$. Suppose $cf(z) = f(z/\ell)$ for some nonzero $c \in \mathbb{C}$ (as functions on the upper half plane). Then $f = 0$.

**Proof.** Observe that $f \in M_k(N, \chi)$ implies $f(z + 1) = f(z)$, and therefore

$$ f((z + 1)/\ell) = cf(z + 1) = cf(z) = f(z/\ell). $$

If $f(z) = \sum_{n \geq 0} a(n)e^{2\pi inz}$ is the Fourier expansion of $f$ at $\infty$, comparing coefficients of $e^{2\pi inz/\ell}$ in the Fourier expansions of both sides of the equality above yields

$$ a(n)e^{2\pi in/\ell} = a(n), $$

for all $n \geq 0$. This implies $a(n) = 0$ whenever $\ell$ does not divide $n$ (because $e^{2\pi in/\ell} \neq 1$).

We now observe that $c^2f(z) = cf(z/\ell) = f(z/\ell^2)$, and the same argument shows that $a(n) = 0$ whenever $\ell^2$ does not divide $n$. Repeating this argument ad infinitum shows that $a(n) = 0$ for all $n > 0$. For $k > 0$ there are no nonzero constant functions in $M_k(N, \chi)$, so we also have $a(0) = 0$, Thus $f = 0$ as claimed. \qed

**References**