These notes recall some standard topological terms we will use (and where there is not a standard definition, to indicate which definition we are using), along with some well known (or easy to prove) facts. They are not meant to be mathematically precise, but we will begin by recalling some precise definitions; see [1] or your favorite topology textbook for details.

Definition 1. A topological space is a set X (whose elements are called points) together with a topology $\mathscr{U}$: a family of open subsets of X including X and the empty set, closed under unions and finite intersections. A continuous map of topological spaces is a map of sets $f: X \to Y$ for which preimages of open sets are open. Taking continuous maps as morphisms yields the category **Top** of topological spaces, a concrete category that is complete and cocomplete, with a unique initial object (the empty set), and final objects that are singletons (these are precisely the sets that admit only one topology, all others admit more than one). Isomorphisms in this category are called homeomorphisms.

Definition 2. Let x be a point in a topological space. A neighborhood U of x is a set whose interior (union of open $V \subseteq U$) contains x. The point x is a limit of $S \subseteq X$ if every neighborhood of x intersects S in some $y \neq x$. A sequence $\{x_n\}$ converges (to the limit x) if every neighborhood of x contains all but finitely many elements of $\{x_n\}$, in which case we write $\{x_n\} \to x$.

Definition 3. A metric on a set X is a function $d: X \times X \to \mathbb{R}_{\geq 0}$ with $d(x,y) = 0 \Leftrightarrow x = y$, $d(x,y) = d(y,x)$, and $d(x,y) + d(y,z) \geq d(x,z)$. Every metric d induces a topology on X defined by the basis of open balls $B_r(x) := \{y : d(x,y) < r\}$, making X a metric space. A Cauchy sequence $\{x_n\}$ in a metric space has the property that for every $\epsilon > 0$ we have $d(x_n, x_{n+1}) < \epsilon$ for all but finitely many n. A metrix space in which every Cauchy sequence converges is complete.

Definition 4. A covering map $f: X \to Y$ is a continuous surjection for which each $y \in Y$ has an open neighborhood V whose preimage $f^{-1}(V)$ is a union of disjoint open $U_\alpha \subseteq X$ on which f restricts to a homeomorphism $U_\alpha \xrightarrow{\sim} V$. When such an f exists, X is a covering space of Y. Two covering maps are equivalent if the differ by a homeomorphism $X \to X$.

Definition 5. A homotopy between continuous maps $f, g: X \to Y$ is a continuous map $h: X \times [0,1] \to Y$ such that $h(x,0) = f(x)$ and $h(x,1) = g(x)$ for all $x \in X$. When such an h exists, f and g are homotopic.

Definition 6. A path is a continuous map $f: [0,1] \to X$ (the image of f is a path in X). Two paths $f_1, f_2: [0,1] \to X$ are path homotopic if there is a homotopy $h: [0,1] \times [0,1] \to Y$ with $f_1(0) = h(0,t) = f_2(0)$ and $f_1(1) = h(1,t) = f_2(1)$ for $0 \leq t \leq 1$. Path homotopy defines an equivalence relation on paths.

Definition 7. Given two paths $f, g: [0,1] \to X$ with $f(1) = g(0)$, their product is the path h defined by $h(t) = f(2t)$ for $t \in [0, 1/2]$ and $h(t) = g(2t-1)$ for $t \in [1/2, 1]$; this product operation is compatible with path homotopy equivalence.

Definition 8. A path $f: [0,1] \to X$ with $f(0) = f(1) = x_0$ is a loop with base point x_0. The fundamental group $\pi_1(X, x_0)$ of a topological space X is the group of path homotopy equivalence classes of loops with base point x_0 (whose identity is the homotopy class of the constant loop $t \mapsto x_0$). If X is path connected and $\pi(X, x_0)$ is trivial for some (equivalently, any) $x_0 \in X$, then X is simply connected. If every $x_0 \in X$ has an open neighborhood U for which the group homomorphism $\pi_1(U, x_0) \to \pi_1(X, x_0)$ is trival, then X is semilocally simply connected.

Definition 9. A simply connected covering space of X is called a universal covering space. If X is path connected and locally path connected, all universal covering spaces of X are equivalent, and if X is also semilocally simply connected, then it has a universal covering space.

terms applied to sets or families of sets	
closed	complement is open
closure	intersection of all closed supersets
interior	union of all open subsets
dense	closure is a superset
neighborhood	set whose interior contains a specified point or set
cover	family of sets whose union contains every point
open cover	cover consisting of open sets
basis	open cover containing a subset of every neighborhood
path	image of a continuous map $[0,1] \to X$
diagonal	the subset $\Delta = \{(x,x)\}$ of $X \times X$

terms applied to topological spaces (including subsets viewed as subspaces)	
discrete	every set is open
indiscrete	only the empty set and whole space are open
T_1	singletons are closed
Hausdorff	pairs of distinct points have disjoint neighborhoods
regular	pairs (x,C) with $x \notin C$ closed have disjoint neighborhoods
compact	every open cover has a finite subcover
limit point compact	every infinite set has a limit point
sequentially compact	every sequence has a convergent subsequence
locally compact	every point has a compact neighborhood
separable	contains a countable dense subset
connected	not the disjoint union of two closed sets
path connected	every pair of points can be joined by a path
locally path connected	basis of path connected neighborhoods
simply connected	a path connected space with trivial fundamental group
totally disconnected	only the empty set and one-point sets are connected
first countable	every point has countable set of neighborhoods that includes a subset of every neighborhood
second countable	has a countable basis
metrizable	admits a metric that induces its topology
completely metrizable	metrizable as a complete metric space

terms applied to morphisms (continuous maps)	
open	sends open sets to open sets
closed	sends closed sets to closed sets
homeomorphism	has a continuous inverse (also bicontinuous, or isomorphism)
local homeomorphism	restricts to a homeomorphism on a neighborhood of every point
diagonal	the map $\Delta\colon X \to X \times X$ sending x to (x,x)
quasiproper	preimage of a compact set is compact
proper	closed and quasiproper
separated	fibers are Hausdorff

terms applied to topologies	
finer	contains (also stronger)
coarser	contained in (also weaker)
subspace	coarsest topology on subsets making inclusion continuous
product	coarsest topology on products making projections continuous
coproduct	finest topology on coproducts making injections continuous
quotient	finest topology on quotients making quotient maps continuous

Warning 1. In algebraic geometry (and in Bourbaki) compact spaces are required to be Hausdorff; the term quasicompact is used for the notion of compactness defined here, and some authors call quasiproper maps proper (adding the closed condition when needed).

The last four terms in the table refer to ways of creating new topologies from old ones:

1. If X is a topological space, any subset $Y \subseteq X$ can be given the subspace topology in which the inclusion map $\iota: Y \hookrightarrow X$ is continuous. The open sets in Y are precisely the inverse images of the open sets in X.
2. If (X_i) is a family of topological spaces, the product set $X := \prod_i X_i$ can be given the product topology in which the projections $\pi_i: X \twoheadrightarrow X_i$ are all continuous. The topology of X is generated by the inverse images of open sets in the X_i.
3. If (X_i) is a family of topological spaces, the coproduct (disjoint union) $X := \coprod_i X_i$ can be given the coproduct topology in which the injections $\iota_i: X_i \hookrightarrow X$ are all continuous. The open sets of X are precisely those sets whose inverse image is open in every X_i.
4. If X is a topological space and $\sim$ is an equivalence relation on the set X, the set $X/\sim$ of equivalence classes can be given the quotient topology in which the quotient map $\pi: X \twoheadrightarrow X/\sim$ that sends elements to their equivalence classes is continuous. The open sets of $X/\sim$ are precisely those sets whose inverse image is open in X.

Below is a running list of easy and/or standard facts. You are welcome to use these on problem sets without further justification.

fact	justification
finite sets are compact	trivial
a set is closed if and only if it contains its limit points	trivial
closed subsets of a compact space are compact	easy
closed/open sets in locally compact Hausdorff spaces are locally compact	easy
a space is Hausdorff if and only the diagonal is closed	easy
in a Hausdorff space limits are unique	easy
compact subsets of a Hausdorff space are closed	easy
a subset of $\mathbb{R}^n$ is compact if and only if it is closed and bounded	Heine–Borel
continuous bijections $f: X \to Y$ with X compact, Y Hausdorff are open	easy
any product connected spaces is connected	easy
any product of totally disconnected spaces is totally disconnected	easy
any product path connected spaces is path connected	easy
any product of Hausdorff spaces is Hausdorff	easy
any product of compact spaces is compact	Tychonoff
finite products of locally compact spaces are locally compact	easy
not all products of locally compact spaces are locally compact	$\mathbb{R}^\omega$
quasiproper maps to locally compact Hausdorff spaces are proper	easy
if Y is compact the projection map $X \times Y \to X$ is closed	Tube lemma
compact $\Rightarrow$ limit point compact $\Rightarrow$ sequentially compact	easy
for metric spaces: compact $\Longleftrightarrow$ limit point compact$\Longleftrightarrow$ sequentially compact	easy
regular Hausdorff second countable $\Longleftrightarrow$ metrizable and separable	Urysohn

References

[1] James R. Munkres, *Topology*, 2nd edition, Pearson, 2018.