18.782 Introduction to Arithmetic Geometry
Course Outline, Spring 2023

Below is a tentative sequence of topics planned for the course. This is still subject to change; it is likely that we will not have time to cover every topic.

1. **Introduction**
   overview of arithmetic geometry, rational points on conics

2. **Finite fields and p-adic integers**
   root-finding over finite fields, inverse limits, discrete valuations

3. **p-adic numbers**
   absolute values, completions, Ostrowski's theorem, Hensel's lemma

4. **Quadratic forms**
   Hilbert symbols, weak and strong approximation, the Hasse-Minkowski theorem

5. **Introduction to algebraic geometry**
   affine and projective varieties, Zariski topology, affine algebras

6. **Projective varieties**
   rational maps, morphisms, products of varieties, completeness

7. **Smooth projective curves and function fields**
   tangent spaces, singular points, valuations, local rings, desingularization

8. **Divisors on curves**
   closed points, linear equivalence, the Picard group, the degree theorem

9. **The Riemann-Roch theorem**
   Riemann-Roch spaces, genus, the canonical divisor

10. **Elliptic curves and abelian varieties**
    elliptic curves, Jacobians of curves, isogenies

11. **Elliptic curves over Q**
    Nagell-Lutz theorem, Mordell's theorem, the Tate-Shafarevich group

12. **Zeta functions**
    Weil conjectures, Honda-Tate theorem