This assignment is due Tuesday, May 12.

**Instructions.** Though collaboration on psets is encouraged, I’d like each of you to do this assignment by yourselves.

1. Let $C$ be a plane curve of degree four with three nodes. Use projection to $\mathbb{P}^1$ from a generic point of the plane to determine the degree of the dual curve $C^*$. 

2. Let $X$ be the affine line $\text{Spec} \mathbb{C}[x]$. We may view $\text{Spec} \mathbb{C}[x_1, x_2]$ as the product $X \times X$. Then a homomorphism $\mathbb{C}[x] \to \mathbb{C}[x_1, x_2]$ defines a morphism $X \times X \to X$ – a law of composition on $X$. Determine the homomorphisms for which the point $o$: $x = 0$ becomes an identity element for that law. (Determining the laws that make $X$ into a group makes a nice problem. In order to keep the pset short, I’m not assigning the rest.)

3. A pair $(f_0, f_1)$ of relatively prime, homogeneous polynomials in $x_0, x_1$ of the same degree $d$ defines a morphism $u : \mathbb{P}^1 \to \mathbb{P}^1$ that maps a point $q$ to $(1, f_1(q)/f_0(q))$ if $f_0(q) \neq 0$ and to $(f_0(q)/f_1(q), 1)$ if $f_1(q) \neq 0$. By inspecting the inverse images of a few points, determine the maps that are injective, and use your result to describe the group of automorphisms of $\mathbb{P}^1$. 

4. The complement $X$ of the point $(0, 0, 1)$ in $\mathbb{P}^2$ is covered by the two standard affine open sets $U^0, U^1$. Use that covering to compute the cohomology $H^q(X, O_X)$. 