

April 7, 2021

18.702 Quiz 4

Take this quiz any time on Friday, April 9.

As always: You are required to do the quiz alone.

You may consult the text, but no other source.

Let $R = \mathbb{Z}[\delta]$, with $\delta^2 = -5$, let p be an integer prime, and let (p) be the principal ideal of R generated by p . Recall that p *splits* if there is a prime ideal P of R such that $(p) = \overline{P}P$.

The problem: There might be integers a, b such that $p = a^2 + 5b^2$. Using the primes $p = 3$, $p = 11$, and $p = 29$ as examples, explain what this would imply for the splitting or not, and in case of splitting, for the prime ideal P .