18.702 Quiz 4

Take this quiz any time on Friday, April 9.
As always: You are required to do the quiz alone.
You may consult the text, but no other source.

Let \( R = \mathbb{Z}[\delta] \), with \( \delta^2 = -5 \), let \( p \) be an integer prime, and let \((p)\) be the principal ideal of \( R \) generated by \( p \). Recall that \( p \) splits if there is a prime ideal \( P \) of \( R \) such that \((p) = P \cap P\).

The problem: There might be integers \( a, b \) such that \( p = a^2 + 5b^2 \). Using the primes \( p = 3 \), \( p = 11 \), and \( p = 29 \) as examples, explain what this would imply for the splitting or not, and in case of splitting, for the prime ideal \( P \).