

**18.702 Problem Set 1**

due wednesday, February 24

1. Don't use characters to work this problem. Just use linear algebra.

Here  $G$  is the dihedral group  $D_n$  of symmetries of an  $n$ -gon, with the usual generators and relations:  $x^n = 1$ ,  $y^2 = 1$ ,  $yx = x^{-1}y$ .

(a) Let  $\rho : G \rightarrow GL(V)$  be a representation of  $G$ , let  $v$  be an eigenvector for  $\rho_x$ , and let  $w = yv$ . Show that the vectors  $v, w$  span an invariant subspace of  $V$ . Therefore, if  $\rho$  is irreducible, its dimension is at most 2.

(b) Determine the one-dimensional representations of  $G$ .

(c) Show that if  $\rho$  is an irreducible representation of dimension 2, the eigenvectors of  $\rho_x$  and of  $\rho_y$  are distinct.

(d) Determine the isomorphism classes of irreducible representations of dimension 2.  
Hint for part (d): It is convenient to begin with a basis of eigenvectors for  $\rho_x$ .

2. Chapter 10, Exercise 4.2 (a group of order 55)

3. Chapter 10, Exercise 4.10 (completing a character table)

4. Chapter 10, Exercise M.1 (vibrations of a molecule)